

Dendrography and Art History: a computer-assisted analysis of Cézanne's *Bathers*.

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Abstract

Using DIAS and DENDRON, computer applications designed for the study of infectious fungi and cancer cells, we forge a connection in color, composition, and theme between Paolo Veronese's *Les Noces de Cana* and Paul Cézanne's *Baigneuse debout, s'essuyant les cheveux*. From this connection, we bring to light a "hidden" compositional structure, heretofore unidentified, in Cézanne's *Bathers* as a series. Using computer-assisted systems such as DIAS and DENDRON allowed us to detect relatedness in these paintings not necessarily visible to the human eye. With DIAS and DENDRON, we generated dendrograms that clustered paintings related in brightness, saturation, complexity, and color. After studying color, we applied DIAS to the study of Cézanne's composition. Because Cézanne expressed Neoplatonic ideals regarding paintings in his conversations and letters, we programmed DIAS to compute axial lines and "golden sections" according to the dimensions of the square or rectangular painting under scrutiny. Using DIAS we identified the golden rectangle for Veronese's *Les Noces de Cana*, Cézanne's *Baigneuse debout, s'essuyant les cheveux* and Ingres' *La Source* (another likely model for *Baigneuse debout, s'essuyant les cheveux*). From the similarities and differences between the three paintings we identified, regarding their use of axial symmetry and the golden section, we have formulated a new approach to "seeing" Cézanne's composition in his *Bather* series. Cézanne's noumenal bodies, formerly perceived as "awkward" and composed with "baffling imbalance," we can now see as "golden."

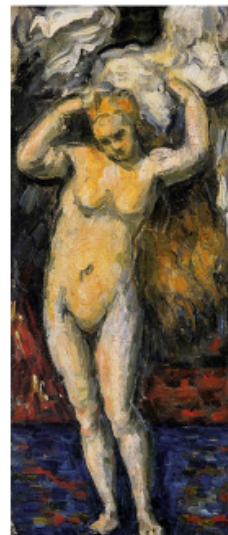


Figure 1. L. Paolo Veronese, *Les Noces de Cana*, 1563, oil on canvas, 22 ft. x 3 in. x 32 ft. Louvre (before conservation) R. Paul Cézanne, *Baigneuse debout, s'essuyant les cheveux*, 1869-71, oil on canvas, 11 3/8 in. x 5 1/8 in. Private Collection. A computer-assisted comparison using the newly developed DIAS/DENDRON ART (DDA) software of fifty-five oil paintings by Paul Cézanne, exhibited at the Galerie Vollard, Paris, in 1895, suggests that Cézanne's *Baigneuse debout, s'essuyant les cheveux*, is highly related to Veronese's *Les Noces de Cana*, in color, saturation, brightness, and complexity (Figures 2-4). Further computer-assisted analysis using DDA suggests that Cézanne drew from Veronese a "hidden" compositional structure to organize his *Bather* paintings

Introduction

Wölfflin V.2

Art history as an academic discipline is relatively new [Bod 2013, 311–325].^[1] Formally established in 1834 at the University of Berlin with the appointment of Franz Kugler (1808-1858) as the first professor of art history, Kugler and his successors Jacob Burkhardt (1818-1897), and Heinrich Wölfflin (1864-1945) instituted methods for the formal analysis of art and specified many categories used today for classifying art [Bod 2013, 316].^[2] Wölfflin is noted for "using parallel projectors in the delivery of art history lectures so that images could be compared."^[3] If art history is relatively new as an academic course of study in the west, computer-assisted analysis of art is in its "infancy," with Wölfflin, aided by slide projectors, as the first scholar to demonstrate the utility of technology in the field.^[4] As the editors of *Digital Art History* note in their 2015 inaugural issue, Wölfflin's mode of comparison "changed the method of art history for good:" "His scientific achievements were a game changer in perceiving, analyzing, and presenting works of art in the scientific world and beyond" [Klinke 2015, 7].

To say that comparisons have produced the categories that structure art history as we know it today is not overstating the case. In "Collections and/of Data: Art

History and the Art Museum in the DH Mode," Matthew Battles and Michael Maizels stress the centrality of the comparative slide lecture in the discipline. They write: "The slide lecture maintains a powerful grip on the collective art-historical imagination; it is the format through which art historical [sic] training begins, and it is by far the most popular means by which the discipline spreads its views and virtues to the mass of uninitiated undergraduates." [5] Lev Manovich, in the first issue of *DAH* concurs: "to explore is to compare. And to compare, we first need to see" [Manovich 2015, 33]. While computer-assisted text analysis has a robust history since the 1950's, the *Proceedings of Computer Vision and Image Analysis of Art* in 2008, 2010 and 2011 are the only journals devoted to exploring computer-assisted methods for art analysis prior to *DAH* in 2015. In the 2008 *Proceedings*, editors David Stork and Jim Coddington propose that computers "can reveal the fallibility in our perception and teach all of us to see with a more informed eye and mind" [Stork 2008]. [6] Computer-assisted methods of analysis can "question the categories we already have, generate new ones, or create cultural maps that relate cultural artifacts in original ways" [Stork 2008, 24]. Since 2008, computer scientists in collaboration with museum curators have applied computational methods to art for conservation, reconstruction, and restoration. X-ray imaging, infrared reflectography, UV-induced fluorescent imaging, and photo-induced luminescence photography can reveal the under-layers of paintings, identify the properties of pigments and binders, authenticate artworks, detect forgeries, and gauge optimal designs for museum and digital displays. Computer algorithms are used to measure and count brushstrokes, sort and classify artworks by style, identify the salient features of an artist's style, and produce three-dimensional models that illustrate an artist's working methods.

Even though digital tools make art available for viewing, pedagogy, and research like never before, [7] the use of data-enabled science, or algorithms, to interpret digitized art and compare painting styles continues to provoke skepticism among humanists. [8] As Stephen Ramsay observes in "Humane Computing": "They fear an epistemology that does not merely value empirical data, but which (in its extreme philosophical forms) considers empirical data to be the only valid form of evidence" [Ramsay 2016]. We believe that the visualizations produced by computer applications do not replace direct experience of the artifact. Correlations do not necessarily indicate causality or an ultimate relationship in every instance. While a computer may place together certain artists with similar styles in a cluster, this does not always imply that there is a historical relationship between them, as their styles may have simply undergone convergent evolution. The computer's lack of historical and artistic context means that the process always has a requirement of human interpretation.

We propose here computer-assisted techniques for comparing color, saturation, brightness and complexity between digitized paintings that updates Wölfflin's method. No studies of digitized art to date use dendrography, a well-established method in computational biology since 1990. DIAS [Soll 1995, 43–104], an image analysis program, and DENDRON [Soll 2000, 332–370], a method for generating trees reflecting relatedness, are patented, tried-and-true software programs. Over 400 citations in peer-reviewed literature for the basic methods paper describing DENDRON support efficacy [Soll 1995]. Dendrography pre-dates the burgeoning field of artificial intelligence and machine learning. The efficacy of the very latest computer cognition systems based in machine learning is viewed at this time as inconclusive. As *Science Daily* (1/7/19) reports in "Can artificial intelligence tell a teapot from a golf ball? Severe limitations of 'deep learning' machines" — even the latest programs cannot accurately identify shape when a pattern associated with another shape is grafted onto it. Machine learning systems "are easily fooled." [9] Dendrography is a traditional computer-assisted method of comparing images and patterns. To that end, we believe that the computer applications DIAS and DENDRON, having been validated in many fields, may reliably "extend the senses," [10] of the art historian. The visualizations they produce suggest similarities between paintings easily missed by human beings whose capacity to compare is limited by conceptual and subjective bias, lack of access to a work, or any number of reasons that make creating interesting comparisons challenging.

Dendrography has enormously productive applications for the comparison of formal and iconographic elements in paintings. Since formal analysis is often viewed as outdated in today's art history, its potential as a subject for computational analysis has been overlooked (except for the few curatorial forays in this direction). As such, our approach not only attempts to see the value of the computational method, but also what DENDRON and DIAS may do to open up in surprising new ways what has been considered a finished part of the field. This may be the first study in digital art history that uses dendrography to cluster similar paintings and that employs computer programs to calculate golden rectangles in paintings for comparative purposes. The ever-growing raft of digitized paintings online, and the continual improvement of digital photography, calls for computer-assisted methods as long as these programs are reliable, the questions posed are appropriate for computer-assisted investigation, and the final interpretations are considered just that, namely interpretations.

Using the newly customized DIAS/DENDRON (DDA) programs (Appendix I), we investigate in Part I a persistent connection between *Les Noces de Cana* by Veronese and Cézanne's first female bather *La Baigneuse debout, s'essuyant les cheveux*. *Les Noces de Cana* is enormous in scale, more than twenty-two feet by thirty-two feet in height and length, while the *baigneuse* is tiny, only eleven inches in height and five inches in width. *Les Noces de Cana*'s subject matter is biblical, Christ transforming water into wine at the marriage feast, while Cézanne's work is prosaic, a woman drying her hair after a bath. A traditional art historian, guided by genre expectations, might conclude that scale and theme obviate potential connections between the works, but using DDA, which normalizes scale, we can grasp surprising connections. Now, using DDA analysis, we can see how Cézanne capitalizes on Veronese's color scheme in his first female *baigneuse*. We may also use DDA to suggest new approaches to old problems. For example, Cézanne painted over eighty bathers in his lifetime, but these paintings continue to mystify scholars. To date, they are mostly interpreted in psychoanalytic terms as representing Cézanne's repressed sexuality and/or aggression.[11] When we saw that Cézanne's *baigneuse* might be intimately connected to *Les Noces de Cana*, we then employed DDA to explore potential similarities in composition. In Part 2, using DDA analysis, we discovered that the composition of *Les Noces de Cana* is based on axial symmetry and the golden ratio (phi). When we apply DDA to identify axial symmetry and the golden ratio to *Baigneuse debout, s'essuyant les cheveux* and the rest of Cézanne's bathers, we find that Cézanne imitates the composition of *Les Noces de Cana* in virtually every bather painting. Based on axial symmetry and the golden section, then, we have formulated a computer-assisted approach to "seeing" Cézanne's composition in his Bather series. Cézanne's bathers, formerly perceived as "awkward" and composed with "baffling imbalance," can now be viewed as "golden" [Danchev 2013, 11]. With DDA, we find that Cézanne is more likely guided by mathematics in designing his bather paintings, rather than by unconscious drives or motives.

This approach is an intervention in the newly established trend within the digital humanities of the "micro-digital humanities." [12] We are not, as Michael Greenhalgh imagines of digital art historians, "throw[ing] data" at the computer while hoping that somehow "the computer will make sense of them" [Greenhalgh]. We are not sifting "the great unknown" [Cohen 2009] for connections between texts or visual images, instead, we are comparing smaller curated datasets in order to measure influence between a master and student, a source and a re-imagining.[13] In this case, we are comparing fifty-five curated digital facsimiles of Cézanne's oil paintings exhibited in Ambroise Vollard's 1895 Salon to Veronese's *Le Noces de Cana*, a source Cézanne identified as influential to him as an artist.

Standing before *Les Noces de Cana*, Cézanne is reported to have said regarding its color:

That is what a painting should give us, a harmonious warmth, an abyss in which the eye can sink, a silent germination. A state of grace in colors... To love a painting, first you have to drink it in this way, in long drafts, lose consciousness; descend with the painter into the dark, complicated roots of objects come up again with colors, bloom with them in the sun. You have to know how to see, to feel, especially in front of a great machine such as the one Veronese built [Doran 2001, 133].

Art Historian Susan Sidlauskas describes Cézanne's experience of the Veronese painting as like "a mystic's fusion with the deity" [Sidlauskas 2004, 12]. Cézanne thought of Veronese's colors as "great noumenal entities, living ideas, creatures of pure reason...with whom we might correspond" [Doran 2001, 124]. By "great machine," we take Cézanne to mean that Veronese's work delivers to the aspiring painter figure studies, compositions, color combinations and more, structures that an artist can imitate.

Quality of digital reproductions

As Philip Ball observes in *Bright Earth*, "all reproductions are approximations" [Ball 2001, 271].^[14] And in regards to *Les Noces de Cana* and Cézanne's bathers, we must ask of any facsimile whether it is "well or badly reproduced" [Latour 2011, 278]. Digital reproductions must be as close to the original as possible. Bruno Latour and Adam Lowe discuss at length *Les Noces de Cana*'s complex "trajectory" as an image. What Cézanne saw between 1861, his first trip to Paris, until his death in 1906 is not the *Les Noces de Cana* that we see now in the Louvre, as conservationist efforts have modified the painting. Currently *Les Noces de Cana* exists in its "original" (though repainted) version at the Louvre, and it exists in a fully "restored" version, based on digitization, housed in its "original" context, in a Benedictine refectory in Venice. The Louvre *Les Noces de Cana* is considerably darker than the Benedictine digital facsimile. The coat of the server depicted in the foreground of the Louvre *Les Noces de Cana* was originally red but was changed to green in 1989. Thus, no version today is exactly what Cézanne saw. For our purposes here, we use a digitized image of the pre-1989 *Les Noces de Cana*, with the tabard in his "original" red coat. This, perhaps begrimed with more dust, is the closest "approximation" to what Cézanne would have seen. However, we demonstrate that DDA grouped *Baigneuse debout, s'essuyant les cheveux* in the same cluster with both renditions of *Le Noces de Cana* in dendrograms (Appendix II).

Choosing Cézanne's paintings for computer-assisted analysis is also complicated. Cézanne left many works unfinished, therefore, we selected for this study paintings exhibited during his lifetime. The paintings exhibited in the 1895 Vollard exhibition is an exemplary dataset for computation, as it was Cézanne's first one-man show, with examples of paintings from every stage of his development. Most important, we can presume Ambroise Vollard, Cézanne's friend and patron exhibited the works as "finished" with the artist's support. For information about Cézanne's paintings, and for digital facsimiles, we relied on John Rewald's printed *Catalogue Raisonné* and Walter Feichenfeldt, Jayne Warman and David Nash's *The Paintings of Paul Cézanne: an Online Catalogue Raisonné* [Rewald 1996]. Supplemental Table I lists all paintings used for this study and their digital and material provenance. All digital images are in the public domain for research and educational use. When there were multiple versions of digitized images on the web with different levels of brightness, saturation, and coloring, we consulted the online *Catalogue Raisonné*, for the best match, or the websites of the museums that housed the work. In many cases, we checked digital facsimiles against original paintings on site in New York, Philadelphia, Chicago, and Detroit, not only for accuracy in color, brightness, and saturation, but also for correct margin between figure and frame for our geometric partitioning.

Generating dendrograms

The software program we have developed to compare digitized paintings is a composite of two patented programs DIAS (US patent #5,655,028 1997) and DENDRON (US patent #5,400,249 1995). The customized program DDA, allows the user to store and sort digitized images of pictures in a data file, measure parameters for comparison and then generate dendrograms, visual phylogenies of relatedness based on one or any combination of parameters. Both DIAS and DENDRON began development in the late 1980's. DIAS was developed to analyze cell morphology, shape, cell movement and the dynamics of populations of cells, and has recently been used to study tumorigenesis in 2D and 3D. Dendron was developed to analyze and compare DNA fingerprints of infectious organisms and has been used to follow the progression of diseases in human populations. In "The Ins and Outs of DNA Fingerprinting the Infectious Fungi," DENDRON is described in terms that we apply here for the comparison of paintings between the same or different painters. The combined programs, including the parameters we have measured and compared, and the generation of dendrograms, are described in Appendix I in more detail, with relevant references. In brief, images of a collection of pictures were scanned into a DDA data file. The DIAS portion of the program then automatically computed nine parameters, which included brightness, saturation, complexity, redness, orangeness, yellowness, greenness, blueness and purpleness. These parameters are described in Appendix I. To assess a similarity coefficient between two pictures for any one of these parameters or amongst any combination of these parameters, each parameter is assessed in a range of 0.00 to 1.00, 0.00 representing no relatedness and 0.01 to 1.00 representing progressively increasing values. The similarity coefficient (S_{AB}) for one or any combination of parameters is then computed between all pairs of paintings in a collection, generating a matrix of S_{AB} 's as demonstrated in Appendix I, Table A. Based on the matrix of S_{AB} 's, a dendrogram (i.e., a phylogenetic tree of relatedness) is then generated by the unweighted pairgroup method based upon the arithmetic averages (UPGMA), originally used by Rohlf [Rohlf 1963], discussed in detail by Sneath and Sokal [Sneath and Sokal 1973] and developed for DENDRON by Schmid et al. [Schmid 1990]. The accuracy of the program was validated by Pujol et al [Lockhart et. al. 1997], with more than 410 citations in the literature reported by Google Scholar just for the methods paper "The Ins and Outs of Fingerprinting for Infectious Fungi" [Soll 2000]. In the generated dendrogram (e.g., in Figures 2 and 3), the nodes (i.e., vertical connections) approximate the hierarchy of similarity. An arbitrary threshold is drawn as a black vertical line through the dendrogram simply is a tool for visualizing the refutation of similarity or clades, related groups and clusters of paintings.

Analysis of relatedness using dendrography

Using DDA, we compared Cézanne's fifty-five oil paintings presented at the Galerie Vollard in 1895 and Veronese's *Les Noces de Cana*. The S_{AB} 's for all DIAS parameters were used in generating the dendrogram in Figure 2. The dendrogram generated in this case could be separated into seven clades, a through f, using an S_{AB} threshold of 0.83. Both Veronese's *Les Noces de Cana*, *Baigneuse debout, s'essuyant les cheveux* and six additional Cézanne paintings were included in clade f (Figure 2). Interestingly, of the three paintings painted in the 1860s by Cézanne after first viewing *Les Noces de Cana*, two of them, *Baigneuse debout, s'essuyant les cheveux* (1869) and *L'orgie de Nabuchodonosor* (1867), resided in clade f, and were highly related. Since it was obvious that the one color assessed by DIAS in the unrestored *Les Noces de Cana* that appeared disproportionately faded was "green", (and since the painting lacks the heavy green coat of the tabard as in post-1989 retouched versions), we generated a dendrogram in which the composite S_{AB} was computed for all parameters but green (see table in the right lower corner of Figure 3). Using a high threshold, 14 clades were distinguished (Figure 3). One clade, b, clustered six pictures, which included *Baigneuse debout, s'essuyant les cheveux* and *Les Noces de Cana* in a subclade with a node of 0.92 and a subcluster of four paintings, three

of which (*Les Begonias*, *La Baignade*, *Nature Morte*) also resided in the clade b, containing *Les Noces de Cana* and *Baigneuse debout, s'essuyant les cheveux*, using all parameters in Figure 2. Since a digital version of the restored *Les Noces de Cana* is widely accessible on the web, we generated a dendrogram using the complete collection of parameters and obtained similar clustering of the restored facsimile of *Les Noces de Cana* and Cézanne's *Baigneuse debout, s'essuyant les cheveux* (Appendix II). Again, Cézanne's *L'Orgie de Nabuchodonosor* clustered with Veronese's *Les Noces de Cana* and Cézanne's *Baigneuse debout, s'essuyant les cheveux*. The two Cézanne paintings were painted in 1867 and 1869, respectively, so both reflect early works potentially highly related to *Le Noce de Cana*. We also analyzed the restored *Les Noces de Cana* facsimile and the Cézanne collection using all parameters but green, and found that the restored facsimile was in a cluster containing Cézanne's *Baigneuse debout, s'essuyant les cheveux* and the other clustered paintings (Appendix II). We generated a dendrogram of the Cézanne collection that included, both the unrestored and restored *Les Noces de Cana*. The *Baigneuse debout, s'essuyant les cheveux* and both unrestored and restored *Les Noces de Cana* paintings clustered, and the cluster contained the same additional paintings in the clusters of the dendrograms generated with either the unrestored *Les Noces de Cana* alone or the restored *Les Noces de Cana* (Appendix II, Figure A, B, C, D). In support of the similarity assessed by DDA in the collection in Figure 2, we generated a dendrogram that included Veronese's *Les Noces de Cana*, Cézanne's *Baigneuse debout, s'essuyant les cheveux*, and 19 paintings by other artists including Delacroix, Rubens and Tintoretto, all of which were shown in the Louvre between 1861 and 1906 (Appendix III, Figure A). The *Les Noces de Cana* and *Baigneuse debout, s'essuyant les cheveux* paintings grouped into a primary cluster, indicated by thick red lines, again indicating not only the similarity between Veronese's *Les Noce de Cana* and Cézanne's *Baigneuse* paintings, but also the accuracy of the DDA programs.

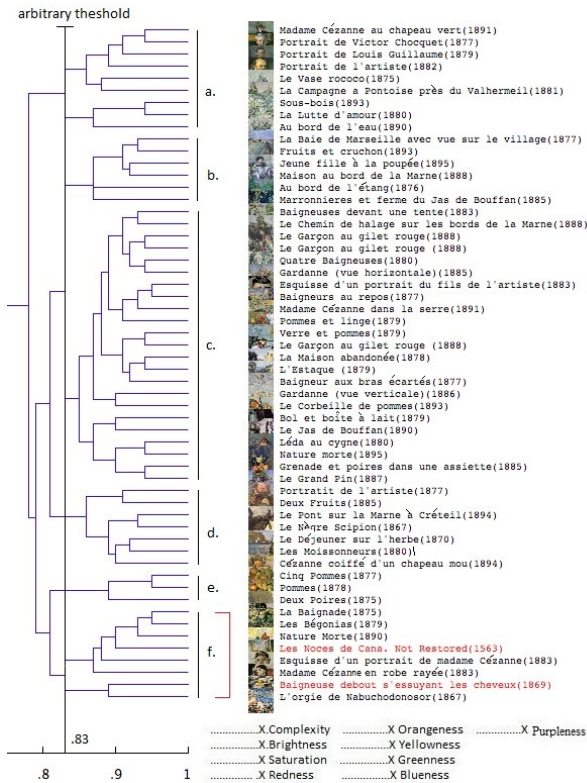


Figure 2. 1895 Galerie Vollard paintings and *Les Noces de Cana*. The dendrogram sorted 56 digitized oil paintings (55 by Cézanne, 1 by Veronese) into clades based on relatedness defined by six parameters. The vertical black line represents an arbitrary threshold (.83) to define moderately related clusters of paintings based on all parameters maximized. These major clades are noted (a through f) by shorter and thinner vertical bars to the right of the dendrogram. Paul Cézanne's *Baigneuse debout, s'essuyant les cheveux* and Paolo Veronese's *Les Noces de Cana* are connected at .83.

In our first experiment (Figure 2), we found potential correlation between the *Les Noces de Cana* and the *Baigneuse*, along with other paintings. In our second experiment (Figure 3), we found that when we control for green, *Les Noces de Cana* and the *Baigneuse* correlate more closely at .9.

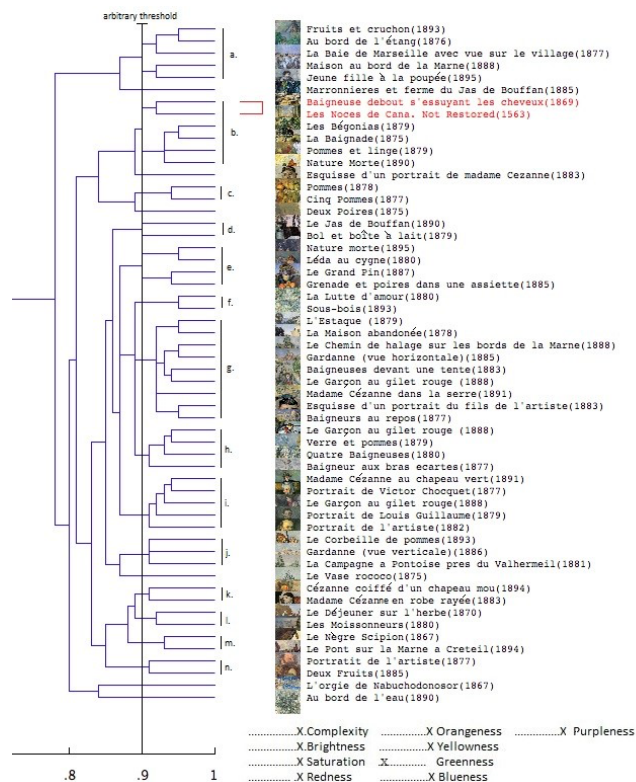


Figure 3. 1895 Galerie Vollard paintings and the unrestored *Les Noces de Cana*, controlled for green.

In Figure 3, *Les Noces de Cana* and *Baigneuse debout, s'essuyant les cheveux* cluster with three other paintings: *Les Bégonias*, *La Baignade*, *Pommes et Linge*, and *Nature Morte* at .9 with *Les Noces de Cana* and *Baigneuse debout, s'essuyant les cheveux* further discriminating as similar at .91.

Part 2. Hidden Patterns

Just like Veronese's *Les Noces de Cana*, Paul Cézanne's paintings continue to inspire wonder in his admirers. His contemporaries, Renoir and Matisse, spoke of his deployment of color and composition in godlike terms, while scholars today "struggle to know" his secret. Cézanne's last words in 1904 to the young painter Emile Bernard heighten the sense of mystery and unknowability that surround the painter's process. Richard Schiff's 2004 Introduction to *Conversations with Cézanne* illustrates this mystery:

"He takes his secret to the grave," remarked Bernard to his wife, upon receiving word of Cézanne's death: "he wrote that he wanted to tell me everything, and I don't know what he meant by that." Regretting his failure to visit Aix in 1906 as planned, Bernard knew that Cézanne, only a month before, had suggested that both men would better "explain themselves" whenever they finally met again in person. Even more poignantly, Bernard could recall the cryptic promise Cézanne included in a letter of the previous year: "I owe you the truth in painting [en peinture], and I will tell it to you" [Doran 2001, xxxiv].

Shiff concludes, "Like Bernard-like Cézanne himself-we struggle to know" (my italics). We propose Cézanne worked from a plan, and that his comments to Bernard indicate guardedness about sharing his process. As he advised the fellow painter Maurice Denis: "You have to have a method. My father [said], 'People have to play games.' That's what I search for in painting...theories, sensations and theories...I wanted to copy nature but I never could do it" [Doran 2001, 93–94]. In this second part of our analysis we identified parameters for DDA which we believed would allow us to begin to assess similarities based on composition. We tested whether *Les Noces de Cana* and *Baigneuse debout, s'essuyant les cheveux* shared potential compositional similarities, and we sought parameters related to composition that could be quantified. Because of Cézanne's frequent references to "harmony" in his conversations and letters, and his documented love for classical Greek and Roman culture, we turned to Classical conceptions of harmony, based on numbers and geometry, for parameters that we could quantify.

In this section, we compare only the relationship between axial lines and golden rectangles as determined by the picture's frame in *Les Noces de Cana* and *Baigneuse debout, s'essuyant les cheveux*. We decided to automate identification of the center and central axis of a painting, because of their technical importance to an artist, and because of their semantic weight. For Rudolf Arnheim, the dead center of a painting is not dead but "alive with tension" [Arnheim 1974, 11, 16]. The vertical and horizontal bisecting lines emanating from the center establish a "solid spatial trellis in the brain." An artist must master the axis first, according to Arnheim, as it "remains the base of reference that alone make obliqueness possible. Obliqueness is always perceived as a deviation [from the axis] hence its strongly dynamic character." Computer-assisted methods for identifying centers and axial lines are helpful as these can be mis-apprehended by a viewer. For example, upon first glance, many people see Christ as the center of *Les Noces de Cana*, as opposed to the men carving meat on the balustrade above him. Once we used DDA to identify the center and the horizontal and vertical bisecting lines of *Les Noces de Cana*, we chose to automate the identification of golden sections in a painting as determined by the square picture plane. In Appendix IV, we present the steps in the DDA program employed to automatically identify a number of geometric landmarks including what we have defined as the "golden window," in a painting. This involves identifying the "golden rectangle" and the axial lines bisecting both vertically and horizontally the picture plane.

Many scholars have attempted to find hidden patterns in art works based on numbers. Most recently, Robert Bork's studies of golden sections in medieval

cathedral ground plans show that for centuries, artists and architects have used the mysterious divine proportion or golden ratio (1.618...) in their designs [Bork 2012]. For Arnheim, the golden ratio is satisfying “because of its combination of unity and dynamic variety, whole and parts are nicely adjusted in strength so that the whole prevails without being threatened by a split, but at the same time, the parts retain some self-sufficiency” [Arnheim 1974]. Veronese and Cézanne would have been familiar with Luca Pacioli’s handbook for artists, *De Divina Proportione*, published in 1509. Seventy-one chapters are dedicated to demonstrating the “golden” section, a mathematical proportion based on an equation that produces “phi” or the never-ending value of 1.618... in artworks [Pacioli 2014]. Cézanne’s contemporary Symbolists, Odilon Redon and Paul Serusier, considered Platonic geometry, holy, or mystical, because it appeared in nature as repeated numbers and patterns. Cézanne and Veronese absorbed the concept of divine proportion in their training, and when they had the problem of creating a two-dimensional structure that expressed “witnessing the holy,” they found a rule or “game” in the golden section that suited their purposes. Using DDA, as a tool to section a painting geometrically, we show that the golden section may have a formal and thematic purpose for Veronese in *Les Noces de Cana* (Figure 4) and that Cézanne might have elaborated on this compositional structure in his Bather paintings.

While we could have discovered this hidden composition by applying a ruler and compass to reproductions from books, the advantage of DDA in studying composition is the mechanization of the geometer’s tools, and the quantity of digital reproductions we can quickly, and more precisely, “rule” than with human hands. It also allows for the quantification of parameters from the digitized image for further comparison using DDA software. DDA determines and saves golden ratio rectangles from any point on a painting, drawing up to nine symmetrical vertical or horizontal staves across a painting, and imposing and saving geometrical shapes on digital paintings. Since these parameters are associated with Neoplatonic ideals and classical harmony, we feel these might have bearing on Cézanne’s painterly concerns in his *Bathers* series. *Baigneuse debout, s’essuyant les cheveux* achieves its uniqueness not only from a masterful deployment of color modelled on Veronese’s *Les Noces de Cana*, but also from its invisible scaffolding, based on Platonic geometry. We believe that Cézanne hides this “game” in plain sight.

Cézanne left eighty *Bather* paintings in various states of completion after his death in 1906 at the age of 68. He repeats this, heretofore unidentified, compositional structure throughout his Bather series. Since Cézanne appears to have modeled his first female bather on *Les Noces de Cana*, as our dendrograms imply, it is likely that Cézanne is linking *Les Noces de Cana* and *Baigneuse debout, s’essuyant les cheveux* semantically. The subject of *Les Noces de Cana* is a wedding feast at Canaan where Christ performs the miracle of turning water into wine, a potent metaphor for the artist translating raw materials into representations. Christ transmutes water, something common, into something holy, strange, and magical; likewise, Cézanne divinizes his bathers; he makes them noumenal, by way of his predecessor Veronese — through color harmonies and divine geometry.

Cézanne viewed geometry as a divine architecture, and he understood the center of a bounded four-sided square to be its axial eye. The center of a painting represents the “truth” or the “eye of god” [Hendrix 2004]. In its aggressively axial construction Figure 4, Veronese appears to take account of the square frame, its center, the golden rectangle, and the golden window in organizing his composition for thematic effect (Appendix IV). The sacrificial meat above Christ’s head is at the exact center of *Les Noces de Cana*. Like a blade, the vertical and horizontal axes divide the meat. The axis signifies that the focus of the painting is sacrifice, and its cognates, Christ, directly below the sacrificial meat, and the dog at the bottom of the painting. The vertical axis bisecting the painting runs directly through Christ’s right eye and the left eye of the dog. As a dog signifies loyalty in medieval/renaissance Christian iconography, the implied vertical line uniting sacrifice/Christ’s eye/loyalty appears to emphasize Veronese’s aim of producing a sensually enlightening painting. To make the painting’s purpose legible, he provides a strong horizontal balcony across the middle of the painting above Christ’s head. The top of the balcony passes through the exact center of the painting.

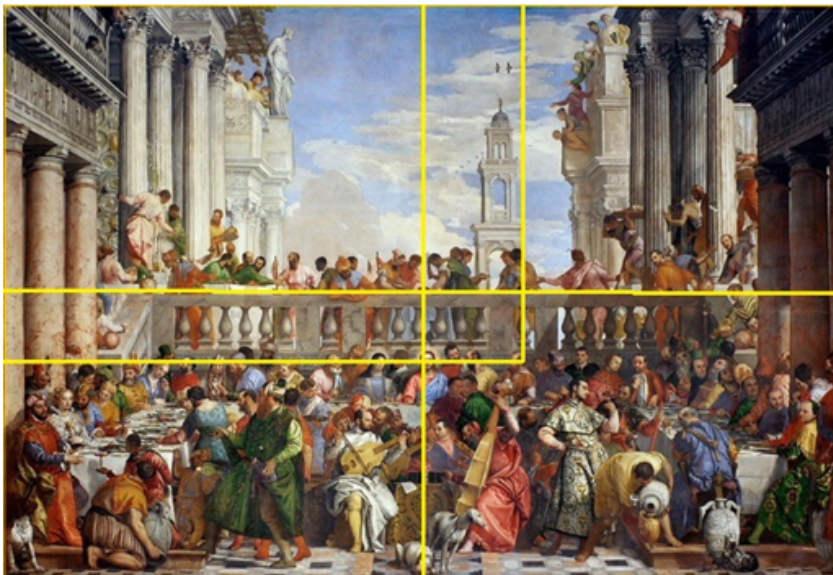


Figure 4. Figure 4. *Les Noces de Cana* (restored version). Golden rectangle, golden window, horizontal and vertical bisecting lines.

In *Les Noces de Cana* Figure 4, just as axial intersection emphasizes theme, so too does the golden section. The golden section intersects Christ’s eyes, and the eyes or mouths of the witnesses flanking Christ. The “golden window” we have identified frames a quarter of Christ’s face, partial faces of three witnesses to his left (specifically through the mouth and eyes). Then, above and behind them, three convex columns supporting the balustrade where butchers are preparing sacrificial meat. Technically, the benefit of a “golden window” for Veronese is that it allows him to represent depth by mediating the distance between flat wall and blue sky with a band of recessing modulations. In the golden window of Figure 4, we glimpse the tablecloth behind the columns of the balustrade above Christ’s head, as well as the robes of the butcher, and a building in the distance. In Figure 4, the golden window underscores theme and creates a sense of depth or of “descended” figures within the composition of the painting. As Christ has “descended” from his godlike nature to be among humans, the golden ratio

becomes the line along which Veronese arranges Christ and his witnesses at the feast. In Figure 5, we see the lowering effect afforded by the golden window. Through the golden window, we see Cézanne's central bather (whose right knee and shoulder are aligned with the vertical axis of the picture frame), descending into a pool of water. We can now say that, in addition to discovering surprising connections in color, saturation, complexity and brightness in paintings, there are landmarks in paintings that DDA automatically computes. These are the center of the painting, the horizontal and vertical bisecting lines (axis), the golden rectangle, the golden strips (the space between the vertical and horizontal bisecting lines and the golden sections of the upper right and lower left sides of the picture plane), and the golden window (located at the right base of the golden rectangle).

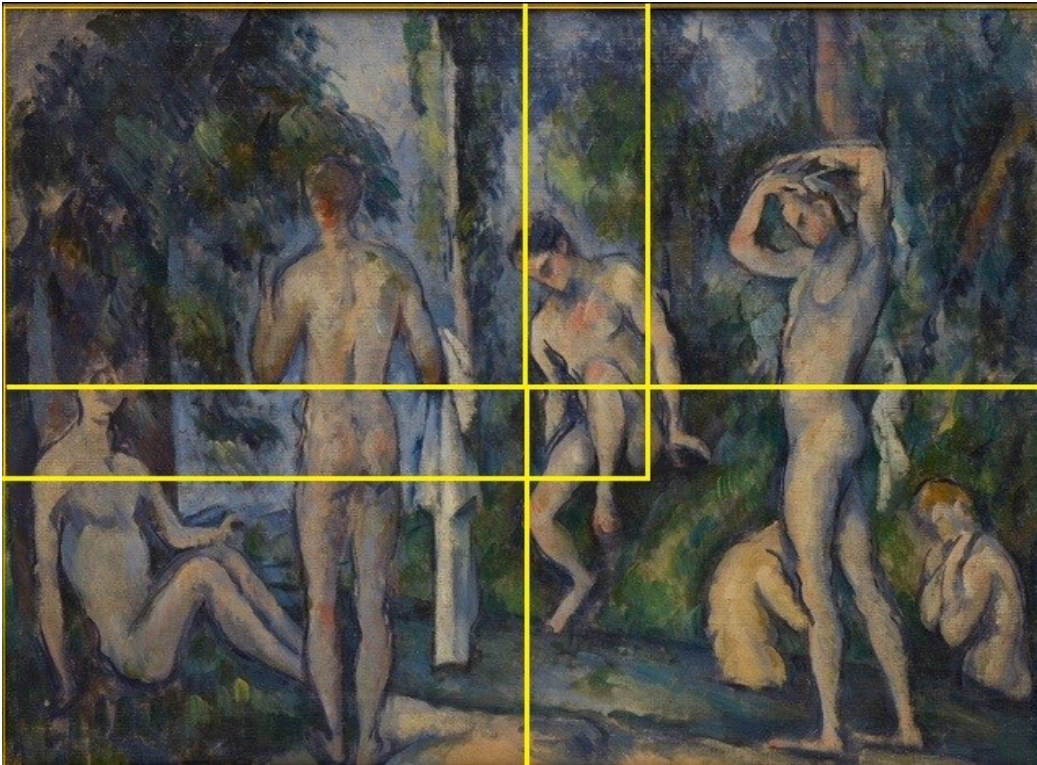


Figure 5. Figure 5. Golden Window: Cézanne signaling its "lowering" effect. Paul Cézanne, *Baigneurs*, 1890, 15 3/8 x 20 7/8in. In addition to being the source as life and the means of ritual purification, water has literary associations with the bucolic poetry of Theocritus and Virgil.

Like Leonardo da Vinci, Cézanne thought paintings were superior to poems because they inspired reverence in viewers [Kemp 1989, 28]. As he expressed to Leo Languier, "Art is a religion. Its goal is the elevation of thought" [Doran 2001, 17]. He wanted to create in paintings a "harmony parallel to nature," a noumenon, in that the painting would be a thing in itself (with a life of its own) whose purpose was to "enlighten" [Doran 2001, 39]. Cézanne stated his commitment to the "law of harmony," numerical patterns he saw in nature, throughout his letters and conversations. He stated: "To read nature is to see it beneath the veil of interpretation as colored patches succeeding one another according to a law of harmony. The major hues are thus analyzed through modulations"" [Doran 2001, 38].

By harmony, Cézanne means harmonia in a Platonic sense of "fitting together:" not only the ideal forms and proportional arrangements of forms based on number found in nature, but also as "the reconciliation of opposites by a third element, bringing them all into a new unity" [Schneider 1994, 236]. Cézanne synthesized contrasts: he "modulated" color as a way of building form. He saw the task of reconciling opposites as a magician's task. He stated: "Painting is not beautiful if the surface plane remains flat. Objects must turn, grow distant, live. That is the magic of our art" [Doran 2001, 66]. To restate, in the golden section, Cézanne finds a way to represent infinity in his paintings, which for him, like the Symbolists, was the essence of nature. In the synthesis of opposites, whether in color contrasts, spatial "near and far," or "up and down," Cézanne creates a "harmonious" painting.

Cézanne and Ingres

Certainly, *Les Noces de Cana* is not the sole model for *Baigneuse debout, s'essuyant les cheveux*. Relevant to our purposes here, a study of Cézanne's Bathers, as a series, from their origin and into their development, Cézanne's ambivalence toward French Neo-Classicism, and its arch-representative Dominique Ingres is well-documented. Cézanne would have seen *La Source* at the Louvre (Figure 6).^[15] *La Source*, begun by Ingres in 1820, and completed in 1856, when Ingres was 76-years-old, was infamous to impressionist painters as its graceful lines and classicism represented the highest standard in Fine Art at the time. Cézanne did not care for the painting. He called it "system and false spirit" [Doran 2001, 132]. Applying axial lines and golden windows to *La Source* and *Baigneuse debout, s'essuyant les cheveux*, we can now see Cézanne copying and departing from his model in surprising ways.

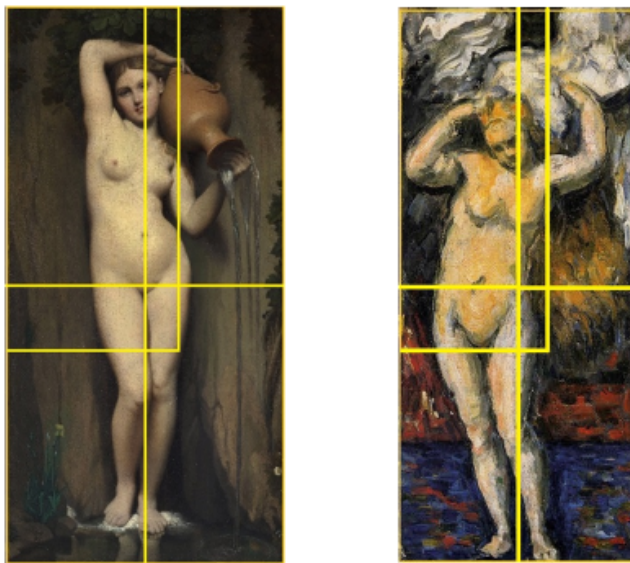


Figure 6. Figure 6. L. Dominique Ingres, *La Source*, 1820-1856, 5'4 x 2'7 in. R. Paul Cézanne, *Baigneuse debout, s'essuyant les cheveux*, 1869-71, 11 3/8 x 5 1/8 in.

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As Michael Baxandall observes, "influence" works in reverse [Baxandall 1985, 59]. Ingres' *La Source* did not "influence" Cézanne. Instead, Cézanne drew a pattern from Veronese and subjected another model, Ingres' *La Source* to the pattern, then dilated upon the results to suit his aims, the "harmonizing" of the youthful nude body. First, we notice that model and copy are mirror images. With arms raised, they form S shaped figures in classical contrapposto. They are symmetrical only in that, when a golden rectangle is formed over the rectangular plane of the canvasses, both figures' left breasts are framed by the middle vertical line and the golden ratio vertical lines formed by the golden rectangle. The middle vertical axis also runs through both figures' left eyes. The golden window in Ingres' work frames the girl's left thigh. In Cézanne, the golden window alludes to the girl's thigh and hip by partially representing (yet with more convexity) her right hip through tonal modulation. Pointedly, Cézanne uses the subordinate horizontal line of the picture plane's golden rectangle to "lower" his figure. In *La Source* the axial eye (picture center) is the sex of the young girl emptying the urn of water. The golden section runs across her legs above her knees. Clearly, in *La Source*, the picture center, as the "eye" of "god," is Ingres' visual pun on the source, water, and female genitalia. In Cézanne's work, the exact center is the nude female's hip swiveled in classical contrapposto, perhaps indicating that his "source" is "classical art," while her sex is aligned with the golden section, making it "golden." In *Baigneuse debout, s'essuyant les cheveux*, by aligning the female sex with the golden section, within a theme that involves washing the body, and in the case of *Baigneuse debout, s'essuyant les cheveux*, a woman drying her hair, Cézanne is "harmonizing" the "studio" eroticism of Ingres' young nude — he is bringing "the source" back to nature.^[16] He also "classicizes" the nude human form by posing his bathers in natural settings, and by associating them with the idealized friendships and homo-social eroticism of the Classical world. In his earliest male bathers as well, Cézanne aligns sex with the golden section of the left side of the frame (Figure 7).



Figure 7. Figure 7: Early Male Bathers: Axiality and Golden Rectangles L. Paul Cézanne, *Baigneur aux bras écartés*, 1876, 9 1/2 x 5 1/2 in. M. Paul Cézanne, *Baigneur aux bras écartés*, 1876, 9 1/2 x 6 3/8 in. R. Paul Cézanne, *Baigneur aux bras écartés*, 1877-1878, 28 3/4 x 23 5/8 in.

In Figure 7, as in female nudes, Cézanne encloses the left breast of an early male bather (7.L.) in the golden rectangle and the vertical bisecting line of the painting. In the middle painting (7.M.) he centers or aligns the left male breast between the golden rectangle and the vertical bisecting line of the painting. In the

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right painting (7.R), the bather's left breast aligns with the vertical bisecting line, while the bather's left elbow aligns with the golden rectangle. In all three works, the golden window captures thigh and background. The navel aligns with the horizontal bisecting line and sex aligns with phi (the bottom side of the golden rectangle determined by the dimensions of the painting).

Golden Windows: Coming into Focus

Eighty Bather paintings exist in various states of completion out of Cézanne's extant oeuvre of 1300 oil paintings and watercolors [Feilenfeldt]. Of the 80 Bathers, 64 are available online for download for research and educational purposes. Using DDA, we identified the axis determined by the picture plane of each digital image, and identified the golden window. In Supplemental Table 2, we provide a snapshot image of all 64 golden windows we collected. Here we present twenty of these images that suggest the "lowering or descending effect" afforded by the golden window. All feature a head, breast or rear end. Most of the 64 paintings have a head, breast, thigh, or rear end framed by the golden window, but some frame the brightest point in the painting, or a faint structure in the far distance.

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Figure 8. Figure 8: Other "Noumenal" bodies. The golden window frames the head, heart, thighs, hips and rear end of the descending or descended body. Row 1: L-R: *Trois Baigneuses*, 1876-1877, 9 1/2 x 12 1/2 in., *Trois Baigneuses*, 1876-1877, 20 1/2 x 21 1/2 in., *Trois Baigneuses*, 1875, 12 x 13 in., *Cinq Baigneuses*, 1879-1880, 13 5/8 x 15 in., *Cinq Baigneuses*, 1880-1882, 23 5/8 x 28 3/4 in., *Baigneurs et Baigneuses*, 1880, 7 1/2 x 8 1/4 in. Row 2: L-R: *Baigneuses devant une tente*, 1883-1885, 24 3/4 x 33 1/8 in., *Baigneurs*, 1890, 23 5/8 x 31 7/8 in., *Baigneurs*, 1890, 15 3/8 x 20 7/8 in., *Baigneurs in Plein Air*, 1890-1891, 21 1/4 x 25 5/8 in., *Baigneurs*, 1892, 8 5/8 x 13 in., *Baigneurs et Baigneuses*, 1890, 8 7/8 x 13 15/16 in., *Groupe de Baigneuses*, 1895, 18 1/2 x 30 1/4 in. Row 3: L-R: *Baigneurs*, 1890-1895, 7 1/2 x 10 1/4 in., *Le Bain*, 1892-1894, 10 1/4 x 15 3/4 in., *Les Grandes Baigneuses*, 1894-1905, 53 1/2 x 75 1/4 in., *Les Grandes Baigneuses*, 1896-1906, 52 3/8 x 81 1/2 in., *Baigneurs*, 1898-1900, 10 5/8 x 18 1/4 in., *Baigneurs*, 1900, 11 7/8 x 17 3/8 in., *Baigneurs*, 1902-1904, 9 x 10 1/2 in.

The Golden Strips

Adjacent to the "golden window" (Appendix IV) are the "golden strips" which elaborate on the point emphasized by the window. The "golden strip" in a bather painting is between the axial and golden ratio horizontal lines or vertical lines of the picture plane (Appendix IV). The golden strips allow for an alternation of frontal and rear nude forms with their sexes aligned with the golden section. As with Veronese's *Les Noces de Cana*, Cézanne aligns the heads of the "descended" figures beneath the middle horizontal line bisecting the picture. For Andre Lohte, Cézanne's genius is in his alternation of foreground and background figures. (What we call Cézanne's "game," he calls Cézanne's "ruse"). Lohte stated of Cézanne: "Genius consists in knowing how to compensate, through constant repetition of each phenomenon, for every thrust into depth, by an equal advance or return. Without this ruse the eye would immediately find the horizon, the distance, without being excited by the successive oppositions which retard its quest for space, and procure for it a subtle pleasure" [Loran 1943, 32]. We can now consider Veronese's *Les Noces de Cana* a masterful example of this phenomenon in fine art painting, a strategy in painting with an endless "trajectory."^[17] In Figure 9, we present the horizontal golden strip of *Quatre Baigneuses* (1888-1890) which shows Cézanne's "alternation of foreground and background" described by Andre Lohte. In Figure 9, and in other Bathers, Cézanne's "successive oppositions" have semantic meaning. For us, this signals Cézanne's reconciliation of opposites in his windows and strips. Thematically, Cézanne reconciles body and mind into a third "noumenal" alliance. He aligns the attentive and communicative heads, hearts, and sexes of his descended and/or risen figures, along the golden section and the axial center of his Bather paintings. In Supplemental Table 3, we present the horizontal "golden strips" of the 64 bathers in this study.

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Figure 9. Figure 9: Golden Window, Golden Strip, and Complete Image. *Quatre Baigneuses*, 1888-1890, 28 3/8 x 36 1/4 in. In *Quatre Baigneuses* Cézanne alternates frontal and rear nudity. He aligns the heads of "descended" bathers with the sex of "risen" bathers along the axial center and horizontal golden section of the picture plane.

Beyond the Church

Cézanne presents an affirmative and benedictory vision of youth and beauty in his Bathers. In *Les Grandes Baigneuses*, a 1906 painting left incomplete when he died, the church in the distance shows that his bathers exist in an intermediate space between the viewer outside the picture plane and the church (Figure 10). The bathers are in a space of their own, determined by the geometry that unites them. When we apply the geometric analysis of DDA to the painting, we see that the golden window frames a reality beyond the church. We feel this illustrates Cézanne's Neoplatonic leanings, and fuels his concept of art as a religion. In his serial use of golden sections to structure his bather series, we see an interest in the infinite, and an understanding of the golden window as affording a glimpse of it.

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Figure 10. Figure 10. Imagining the infinite. In *Les Grandes Baigneuses* (1906), we see in the golden strip, frontal and rear nudity, descended heads (frontal, side and rear views), brightest points, and far distance.

Homage to the Classical World

Cézanne also uses sacred geometry to structure the imagined space of the image. In *Baigneurs en Plein Air* (Figure 11), an unfinished painting from 1890-1891, the ground of the five figures forms a Pentagon, below the horizontal line that bisects the painting.

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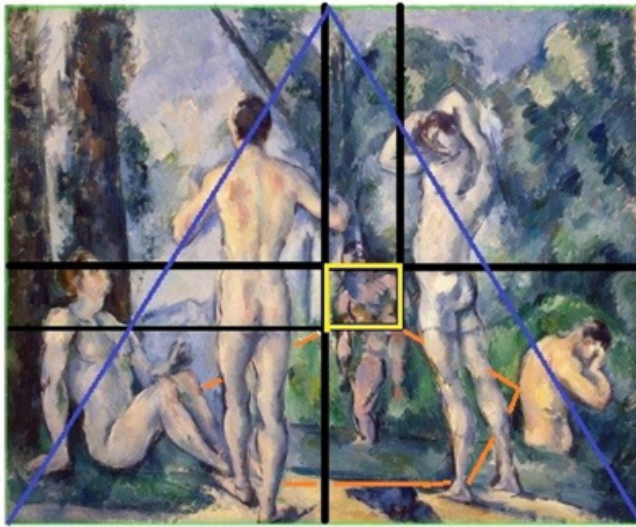


Figure 11. Figure 11: *Baigneurs en Plein Air*, 1890-1891. Five figures form a pentagon in the imagined space of the painting.

As stated in Appendix IV, DDA will discriminate a variety of geometric shapes within a picture. The figure emerging in the golden window represents its triangular point. This implied pentagonal shape illustrates Cézanne's advice to artists throughout his letters and conversations: "Interpret nature in terms of the cylinder, the sphere, the cone; put everything in perspective, so that each side of an object, a plane, recedes toward a central point" [Doran 2001, 29]. In addition to an implied pentagon, these bathers form an imaginary cone, a large triangle parallel to the surface plane whose tip coincides with the middle vertical axis of the paintings. This triangle provides vertical armature for the five bodies, linking them together in one unit with the armpits aligned at the diagonal lines. This compositional triangle both dynamically lifts the figures, and stabilizes the composition.

Cézanne suggests with a few blue and red strokes the figure whose breast is framed in the golden window of Figure 11. Cézanne repeated this particular composition in his search for a realization commensurate with his ideal. We can now think of a man lowering himself into a body of water as a compositional "problem" Cézanne was trying to solve in his Bather series. Painted in four-sided constructive strokes, and in Veronesian, "noumenal" blue and red color patches that resemble a harlequin's costume, the figure is in motion, and not fully formed. He is an intermediate being, not yet concretized, nor fully translated between here and there within the noumenal square of the painting. Cézanne is clearly attuned to magical conceptions of the number four as representing the fourth element of earth in Platonic doctrine and inherent in the squares of the harlequin's costume. He was also aware of the five-sided figure of the pentagon as representing the regenerative aspects of creation. The Harlequin's checkerboard costume has been described as symbolizing "a chthonic force" [Schneider 1994]. If so, this chthonic force's heart is framed in the golden window, a highly significant spot, charged with meaning, as we now see in Cézanne's Bathers, and particularly in this painting (Figure 11) depicting five men in geometrical relationship to each other. In addition to a sacred understanding of numbers and shapes, three (triangle: divinity), four (square: earth) and five (pentagon: creation and regeneration), a means to create a harmony "parallel" to nature, this painting's composition is Cézanne's special homage to the pastoral poetry of Theocritus and Virgil, the classical literature he loved. It is well-known that Cézanne translated Theocritus and Virgil as a pastime in his youth [Doran 2001, 181]. The left side of Figure 11 appears to represent the sympotic aspects of male fraternity with the seated figure under the pine tree discoursing with the standing figure seen from behind. The right side of the painting pays homage to the erotic aspects of homo-sociality implied by Theocritus and developed in the bucolic poems of Virgil. In the poems, herdsmen gather to sing songs about "Daphnis," a young man who drowns himself out of erotic frustration, or his cognates in the classical tradition who die: Hylas, Delphis, Adonis, and Narcissus, other beautiful boys, like the one in Figure 11 "posing" in the foreground. Theocritus and Virgil's lost boys are the poets' reflections on their own lost youth and/or the loss of specific male friends, or lovers [Weinstein]. For Cézanne, "Nature is not on the surface; it is in depth," hence, the man in the water (in the depths) crying on the far right [Doran 2001]. Cézanne's effort to divinize the nude human body in company with other divinized bodies, whether "co-ed" or segregated by sex, and in the many paintings where difference in sex is not explicit, we see the artist's engagement with Neoplatonism, in that his goal is to create a harmoniously simplified world with a playful, affirmative, benedictory vision of youthful nudity.

Conclusion

In this paper, using the automated tools afforded by DDA, we have forged a connection in color, composition, geometry, and theme between *Les Noces de Cana* and *Baigneuse debout, s'essuyant les cheveux*, and from this connection, we have brought to light unifying characteristics in Cézanne's Bathers heretofore unidentified. We have identified the connection between *Les Noces de Cana* and *Baigneuse debout s'essuyant le cheveux* by producing dendrograms of Cézanne's fifty-five 1895 Galerie Vollard Exhibition oil paintings compared to Veronese's *Les Noces de Cana*, a painting we know Cézanne admired at the Louvre during his formative years as a painter and throughout his life. Using computer-assisted methods we generated dendrograms that clustered paintings related in color, saturation, brightness and complexity. After studying these parameters, we applied the geometric program of DDA to the study of Cézanne's composition. Because Cézanne expressed Neoplatonic ideals regarding paintings in his conversations and letters, we programmed DDA to compute axial lines and golden rectangles according to the dimensions of the square, rectangle, and other geometric parameters to the painting under scrutiny. Using DDA we identified the golden rectangle for *Les Noces de Cana*, *Baigneuse debout, s'essuyant les cheveux* and Ingres' *La Source* (another likely model for *Baigneuse debout, s'essuyant les cheveux*). From the similarities and differences between the three paintings identified, based on axial symmetry and the golden section, we have formulated an approach to "seeing" Cézanne's composition in his Bather series. Cézanne's noumenal bodies, formerly perceived as "awkward" and composed with "baffling imbalance," can now be viewed as "golden" [Danchev 2013]. These are only the first parameters, customized to the study of paintings that we have introduced to DDA. As we continue to expand and use this computer program to compare influence between painters, we will add new parameters such as measurements for convexity, concavity, texture and composition, based on edge detection technology in the original DIAS

program. These tools will assist us in investigating a variety of questions basic to art history.

Acknowledgments

This research began with an AAC&U TIDES (Teaching to Increase Diversity and Equity in the Classroom) grant that supported applying technology to the study of paintings from 2013-2017 at Lawrence Technological University. A Howard Hughes Medical Institute's Inclusive Excellence Initiative grant (52008705) is sustaining LTU's integration of computer-assisted analysis of art, music, and literature into its undergraduate curricula until 2021. The Developmental Studies Hybridoma Bank, an NIH National Resource, and the W.M. Keck Dynamic Image Analysis Facility housed at the University of Iowa provided the computational resources for this project. We thank LTU's Research Support Services for providing a Faculty Seed Grant for travel, research assistants Liam Butler, Mary Cody, Irene Missler, and Neil Fraylick. We thank Dean Hsiao-Ping Moore and Prof. Lior Shamir for guidance and support, and LTU Provost Maria Vaz for approving a sabbatical leave Spring 2018, to conduct this work at the W.M. Keck Dynamic Image Analysis Facility at the University of Iowa.

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Appendix I. Analyzing a collection of paintings using the customized DIAS/DENDRON-ART (DDA) program

The newly developed DIAS/DENDRON-ART (DDA) program integrates the 2D image analysis program DIAS (Soll et al., US patent #5655028; Soll, D.R. 1995. The use of computers in understanding how cell crawl. *Int. Rev. Cytol.* 163: 43-104; Soll, D.R. and Wessels, D. (eds.) 1998. Motion Analysis of Living Cells John Wiley, Inc., New York, 298 pages; Kuhl, S., Voss, E., Scherer, A., Lusche, D.F., Wessels, D., Soll, D.R. 2016. 4D tumorigenesis model for quantitation of coalescence, identification of unique cell behaviors and testing anti-cancer drugs. *Chemotaxis Methods in Molec. Biol.* 229-250), and the dendrography software program DENDRON. Schmid, Jan, Edward Voss and David R. Soll. "Computer-Assisted Methods for Assessing Strain Relatedness in *Candida albicans* by Fingerprinting with the Moderately Repetitive Sequence Ca3." *Journal of Clinical Microbiology*. Vol 28.6 (June 1990). 1236-1243. Soll, D. R. "The ins and outs of DNA fingerprinting the infectious Fungi." *Clinical Microbiology Reviews*. Vol.13.2 April 2000. P.332-370. The programs originally written in C++ have been converted to JAVA (Graphics Programming With Java, Second Edition. 2001. Steven's Scott/Jones, Inc.). Images of paintings are scanned into a DDA folder, with borders equal to the inner edges of the frame and normalized for size. If a painting is faded, hue will not be significantly affected since DDA employs HSB (hue, saturation, brightness) color space (Oracle Java 7 documentation, online for "Class Color"), as opposed to RGB (red, green, blue). DDA then automatically computes nine parameters that include brightness, saturation, complexity, redness, orangeness, yellowness, greenness, blueness and purpleness. Additional parameters based on DIAS edge detection programs are now under development and will be incrementally added. The "brightness" parameter is computed by converting the pixels of the entire image to HSB. A value of 0% is black and 100% is white. The average % for the entire picture is computed and the value converted to a range of 0.00 to 1.00. The "saturation" parameter is treated in a similar fashion. It represents the average richness of all colors assessed, is computed from 0 to 100%, then normalized to a value in the range of 0.00 to 1.00. For the "complexity" parameter, the image is first converted to grays with pixel intensities ranging from 0 to 255. The average value is then normalized to 0.00 to 1.00. The final parameters are six hues. On the color or "hue" wheel, they vary from 0 to 360°. The number of pixels with redness at 0° ± 5°, orange at 20 ± 5°, yellowness at 60 ± 5°, greenness at 120 ± 5°, blueness at 240 ± 5° and purpleness at 270 ± 5°, are computed and divided by the total number of pixels in the image. These individual pixel proportions are then converted to 0.00 to 1.00. The values of each parameter, all converted to a range of 0.00 to 1.00, were then used to generate a similarity coefficient (S_{AB}) between every pair of paintings in a collection, using the following formula:

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$$S_{AB} = \frac{\sum_{i=1}^k (a_i + b_i - |a_i - b_i|)}{\sum_{i=1}^k (a_i + b_i)}$$

Figure 12.

where a_i and b_i are the values of a parameter i for paintings A and B respectively, and k is the number of parameters. This formula was derived from that developed by Schmid, J., Voss, E. and Soll, D.R. 1990. (*J. Clin Microbiol.* 28, 1236-1243) for banding patterns of DNA fingerprints in which each band is a restrictive fragment length polymorphism (RFLP) pattern representing a parameter (Soll et. al. 1997 US Patent #5, 655, 028). An S_{AB} of 0.00 represents complete dissimilarity, an S_{AB} of 1.00 identicalness, and S_{AB} s of 0.01 to 0.99, increasing levels of similarity. For this study the average of the parameters of paintings are used to generate the S_{AB} s between all paintings in a collection, which are logged into a matrix, as presented for 18 pictures in Appendix I Table 1.



Figure 13. APPX. 1. Table 1. S_{AB} matrix for 18 pictures in collection

The matrix is then used to generate a dendrogram using the unweighted pair group method based on arithmetic averages (UPGMA), first employed by Rohlf, 1963 Classification of *Aedes* by numerical taxonomic methods [Diptera, Culcidae]. (Ann. Entomol. Soc. Am. 56, 798-804) and discussed in more detail for application by Snesth and Sokel (1973. Numerical Taxonomy W.H. Freeman and Co., San Francisco, CA). In generating a dendrogram, connections are identified in descending values. The matrix is first scanned for the S_{AB} with the highest value. The first two are joined by a vertical node at that S_{AB} . If two or more pairs have the same highest S_{AB} , one is picked randomly to initiate the process of generating a tree. The system then grows the tree of relatedness, connecting pairs, individual paintings and groups of two or more, or groups. The node involving more than two paintings is computed as the arithmetic mean of the S_{AB} s between the pictures connected. This process continues until all pictures are connected to form a relatedness tree (dendrogram). The accuracy of the dendrogram has been assessed by goodness of fit (i.e., whether the distance in the dendrogram approximates the S_{AB} s in the matrix (Avis, J.C. 1994. Molecular numbers, natural history and evolution. Chapraon and Hull, New York, NY). By using completely different methods to assess genetic relatedness between the same collection of strains of an infectious fungus, the accuracy of the DENDRON program was previously verified (Pujol, C., Joly, S., Lockhart, S.R., Sebastian, N., Tibayrenc, M., and Soll, D.R. 1977. J. Clin Microbiol, 2348-2358; Soll D.R. 2000. Clin Microbiol Rev 13, 332-370). The robustness of DIAS/DENDRON ART (DDA) in clustering was also verified by the addition of the restored *Les Noces de Cana* to the Cézanne collection and the unrestored *Les Noces de Cana*, as described in Appendix II.

A comparison of the Cézanne Galerie Vollard collection with the unrestored (Appendix II A) and restored (Appendix II B) versions of Veronese's *Les Noces de Cana*, using all parameters but green. Greenness was removed because in the restored version, a man's cloak, which was red, was converted by restoration to green. Both Cézanne's *Baigneuse debout, s'essuyant les cheveux* and Veronese's *Les Noces de Cana* reside in the same cluster with the restored and unrestored *Les Noces de Cana*. It should be noted that the alternative clusters in the three dendrograms share a majority of other related Cézanne pictures, as is evident in lists of the paintings in the alternative clusters, presented in Appendix II D. These results also demonstrate the accuracy of the clustering method of DIAS/Dendron, given that the addition of different versions, in this case the unrestored and restored *Les Noces de Cana*, or the addition of both, does not disrupt relationships, even though each dendrogram is generated independently, using the unweighted pair-group method employing arithmetic averages, as described in Appendix I.

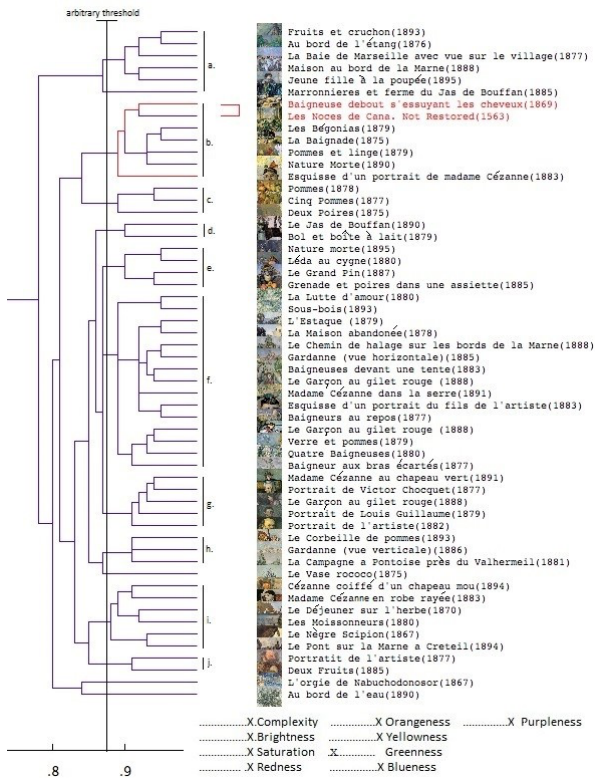


Figure 14. Appendix II. A

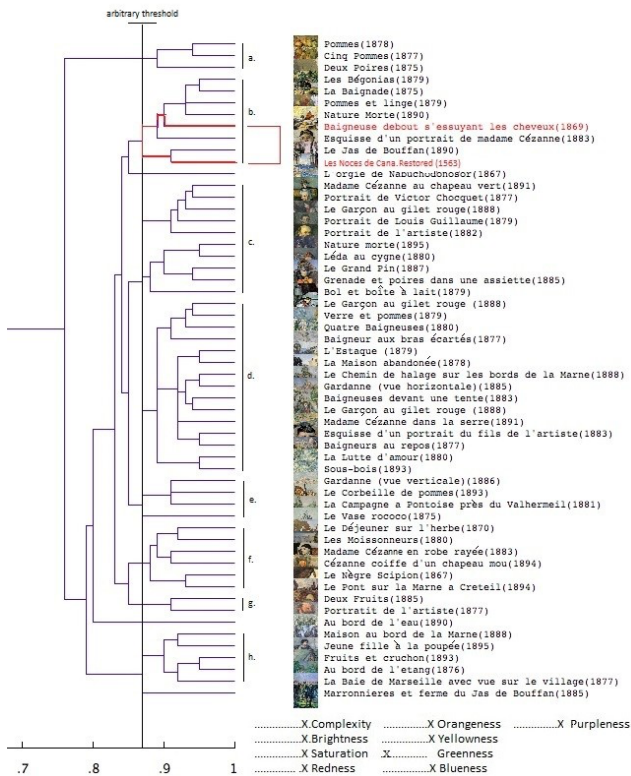


Figure 15. Appendix II. B

Appendix II. C

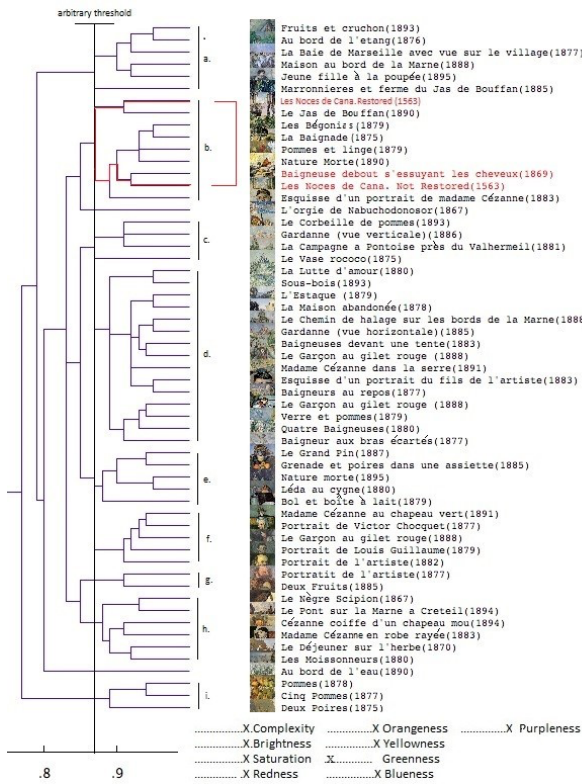


Figure 16.

Appendix II. D

Comparison of pictures in the resident clusters of Cézanne's *Baigneuse debout, s'essuyant les cheveux* and either the unrestored or restored versions of Veronese's *Les Noces de Cana*. Only the first word of the titles of the paintings are included.

Unrestored (Louvre) <i>Cana</i>	Restored (Venice) <i>Cana</i>	Unrestored and Restored <i>Cana</i>
Baigneuse	Baigneuse	Baigneuse
Les Bégonias	Les Bégonias	Les Bégonias
La Baignade	La Baignade	La Baignade
Pommes	Pommes	Pommes
Nature Morte	Nature Morte	Nature Morte
Esquisse de M. Cézanne	Esquisse de M. Cézanne	Esquisse de M. Cézanne
	La Jas de Bouffan	La Jas de Bouffan

Table 1. Appendix II. D

Appendix III. Further demonstration of clustering Veronese's *Les Noces de Cana* and Cézanne's *Baigneuse debout, s'essuyant les cheveux*

To further demonstrate the similarity of Veronese's *Les Noces de Cana* and Cézanne's *Baigneuse debout, s'essuyant les cheveux*, we generated a dendrogram based on the S_{AB} s between these two paintings and nineteen paintings Cézanne would have viewed in the Louvre between 1861 and 1906. The artists include Delacroix (green), Rubens (black), and Tintoretto (blue). *Les Noces de Cana* and *Baigneuse debout, s'essuyant les cheveux* again clustered, supporting the relationship and accuracy of the dendrography program.

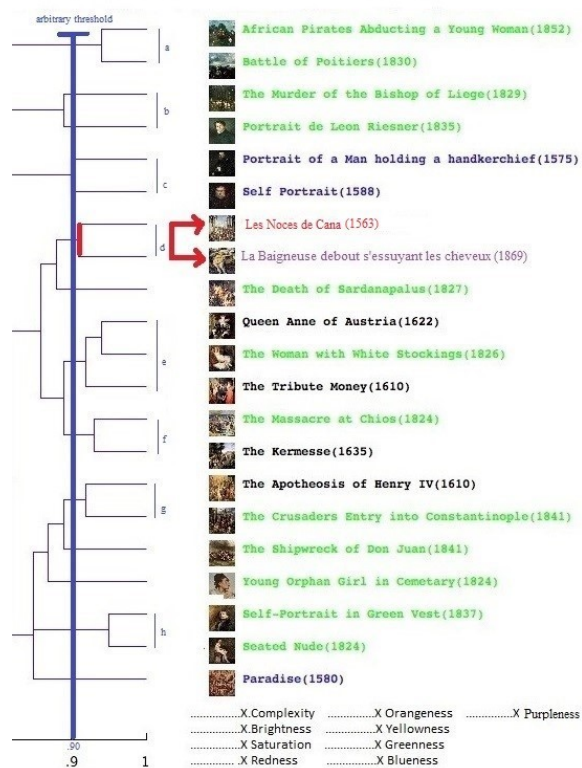


Figure 17.

Appendix IV. How to identify a golden window

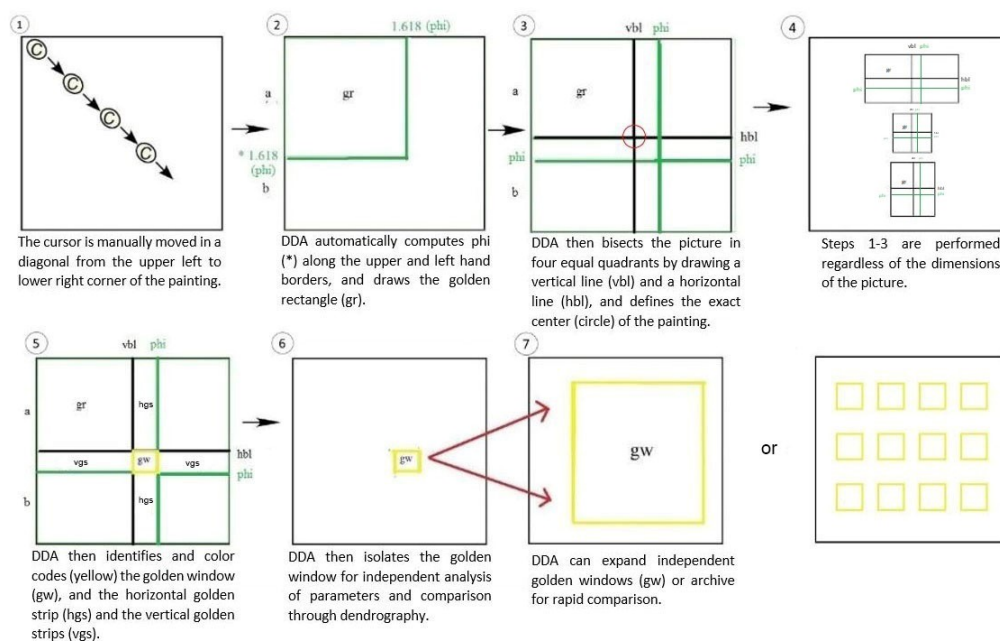


Figure 18.

Supplementary Table I: List of Works

Name of Artist	Title of Work	Medium	Size	Year	Location	Link to artwork used for analysis
Paul Cézanne	La Maison abandonnée	Oil on Canvas	19 5/16 x 23 in	1878-79	Private Collection, New York	https://uploads7.wikiart.org/images/paul-cezanne/
Paul Cézanne	La Baie de Marseille,	Oil on Canvas	26 3/16	1877-79	Vollard Archives,	https://www.moma.org/collection/works/80302?locale=en

	avec vue sur le village		x 32 11/16 in.		Photo No. 604	
Paul Cézanne	L'Estaque	Oil on Canvas	31 1/2 x 39 1/16 in.	1879-83	The Museum of Modern Art, New York	https://www.moma.org/meetme/artwork/index
Paul Cézanne	La Campagne à Pontoise, près du Valhermeil	Oil on Canvas	36 3/8 x 28 7/8 in.	1881-82	Fondation Sirina, Vaduz	http://www.cezannecatalogue.com/catalogue_images/main
Paul Cézanne	Marronniers et ferme du Jas de Bouffan	Oil on Canvas	25 13/16 x 32 in.	1885	Private Collection, New York	https://uploads1.wikiart.org/images/paul-cezanne/
Paul Cézanne	Gardanne (vue verticale)	Oil on Canvas	31 1/2 x 25 5/16 in.	1886	The Metropolitan Museum of Art, New York	http://images.metmuseum.org/CRDImages/ep/original/DT1941.jpg
Paul Cézanne	Gardanne (vue horizontale)	Oil on Canvas	25 X 39 in.	1885	Barnes Foundation, Philadelphia (BF917)	http://www.cezannecatalogue.com/catalogue_images/
Paul Cézanne	Le Grand Pin	Oil on Canvas	33 1/16 x 36 3/16 in.	1887-89	Museu de Arte de São Paulo Assis	https://uploads2.wikiart.org/images/paul-cezanne/the-great-pine-1889.jpg
Paul Cézanne	Maison au bord de la Marne (l'Île Machefer)	Oil on Canvas	28 11/16 x 35 13/16 in.	1888-94	White House Collection, Washington, D.C.	http://www.cezannecatalogue.com/catalogue_images/
Paul Cézanne	Le Chemin de halage sur les bords de la	Oil on Canvas	24 13/16 x 31 1/8 in.	1888	Art Gallery of New South Wales, Sydney	https://www.artgallery.nsw.gov.au/media/collection
Paul Cézanne	Le Jas de Bouffan	Oil on Canvas	29 3/8 x 21 1/2 in.	1890-94	Private Collection, Japan	https://uploads2.wikiart.org/images/paul-cezanne/jas-de-bouffan-1887.jpg
Paul Cézanne	Au bord de l'eau	Oil on Canvas	28 11/16 x 36 3/16 in	1890-92	National Gallery of Art, Washington, D.C.	https://images.nga.gov/en/search/do_quick_search.html?q=%221972.9.1%22
Paul Cézanne	Le Pont sur la Marne à Créteil	Oil on Canvas	28 x 35 3/8 in.	1894	Pushkin Museum, Moscow	https://uploads0.wikiart.org/images/paul-cezanne/bridge-over-the-marne-
Paul Cézanne	Sous-bois	Oil on Canvas	45 11/16 x 31 7/8 in.	1893-94	Los Angeles County Museum of Art	https://collections.lacma.org/node/172409
Paul Cézanne	Le Nègre Scipion	Oil on Canvas	42 1/8 x 32 11/16 in.	1867	Museu de Arte de São Paulo Assis	https://uploads4.wikiart.org/images/paul-cezanne/the-negro-scipio-1867.jpg
Paul Cézanne	Portrait de Victor Chocquet (buste)	Oil on Canvas	13 13/16 x 10 11/16 in	1877	Virginia Museum of Fine Arts, Richmond	https://uploads6.wikiart.org/images/paul-cezanne/
Paul Cézanne	Portrait de l'artiste	Oil on Canvas	10 3/16 x 5 7/8 in.	1877	Musée d'Orsay, Paris	https://uploads7.wikiart.org/images/paul-cezanne/self-portrait-3.jpg
Paul Cézanne	Portrait de Louis	Oil on Canvas	22 x 18	1879-80	National Gallery of Art,	https://images.nga.gov/en/search/do_quick_search.html?q=%221963.10.101

	Guillaume		1/5 in.		Washington, D.C.	
Paul Cézanne	Portrait de l'artiste	Oil on Canvas	18 1/8 x 15 in.	1882	Pushkin Museum, Moscow	http://www.arts-museum.ru/data/fonds/europe_and_america
Paul Cézanne	Madame Cézanne en robe rayée	Oil on Canvas	22 3/16 x 18 1/5 in.	1883-85	Yokohama Museum of Art	https://uploads5.wikiart.org/images/paul-cezanne/
Paul Cézanne	Esquisse d'un portrait de madame Cézanne	Oil on Canvas	8 x 5 1/2 in.	1883	Richard Gray, Chicago	https://uploads5.wikiart.org/images/paul-cezanne/
Paul Cézanne	Esquisse d'un portrait du fils de l'artiste	Oil on Canvas	7 11/16 x 4 1/2 in.	1883-85	Von der Heydt Museum, Wuppertal	http://www.cezannecatalogue.com/catalogue_images/main_lg/R534.jpg
Paul Cézanne	Madame Cézanne au chapeau vert	Oil on Canvas	38 1/2 x 31 1/2 in.	1891-92	Barnes Foundation, Philadelphia (BF141)	https://uploads8.wikiart.org/images/paul-cezanne/
Paul Cézanne	Le Garçon au gilet rouge	Oil on Canvas	31 7/8 x 25 5/8 in.	1888-90	The Museum of Modern Art, New York	https://www.moma.org/collection/works/79086?locale=en
Paul Cézanne	Le Garçon au gilet rouge	Oil on Canvas	31 5/16 x 25 3/16 in.	1888-90	Foundation E.G. Bührle Collection, Zürich	https://uploads5.wikiart.org/images/paul-cezanne/boy-in-a-red-vest-1889.jpg
Paul Cézanne	Le Garçon au gilet rouge	Oil on Canvas	36 3/16 x 28 11/16 in.	1888-90	National Gallery of Art, Washington, D.C.	https://images.nga.gov/en/search/do_quick_search.html?q=%221995.47.5%2
Paul Cézanne	Madame Cézanne dans la serre	Oil on Canvas	36 3/16 x 28 11/16 in.	1891-92	The Metropolitan Museum of Art, New York	http://images.metmuseum.org/CRDImages/ep/original/DP317780.jpg
Paul Cézanne	Cézanne coiffé d'un chapeau mou	Oil on Canvas	23 5/8 x 19 5/16 in.	1894	Bridgestone Museum of Art, Tokyo	https://uploads0.wikiart.org/images/paul-cezanne/
Paul Cézanne	Jeune fille à la poupée	Oil on Canvas	36 3/16 x 28 11/16 in.	1895	Private Collection, New York	https://uploads5.wikiart.org/images/paul-cezanne/young-girl-with-a-doll.jpg
Paul Cézanne	Le Festin – L'Orgie; Le Banquet de	Oil on Canvas	51 3/16 x 31 7/8 in.	1867	Private Collection, France	https://uploads4.wikiart.org/images/paul-cezanne/the-feast-the-banquet-of
Paul Cézanne	Le Pêcheur à la ligne	Oil on Canvas	10 5/8 x 14 3/16 in.	1868-70	?	Unable to locate color version
Paul Cézanne	Le Déjeuner sur l'herbe	Oil on Canvas	23 5/8 x 31 7/8 in.	1870	Private Collection, New York	https://uploads0.wikiart.org/images/paul-cezanne/luncheon-on-the-grass-
Paul Cézanne	Au bord de l'étang	Oil on Canvas	17 1/2 x 21 1/16 in.	1876-77	Museum of Fine Arts, Boston	http://www.mfa.org/collections/object/download/50439
Paul Cézanne	Le Déjeuner sur l'herbe	Oil on Canvas	13 3/8 x 15	1878-80	?	Unable to locate color version

			3/8 in.			
Paul Cézanne	La Lutte d'amour	Oil on Canvas	14 7/8 x 18 5/16 in.	1880	National Gallery of Art, Washington, D.C.	https://images.nga.gov/en/search/do_quick_search.html?q=%221972.9.2%22
Paul Cézanne	Les Moissonneurs	Oil on Canvas	10 3/16 x 16 1/8 in.	1880	Private Collection, Switzerland	http://www.cezannecatalogue.com/catalogue_images/main_lg/454.jpg
Paul Cézanne	Léda au cygne	Oil on Canvas	23 x 28 7/8 in.	1880	Barnes Foundation, Philadelphia (BF36)	https://uploads0.wikiart.org/images/paul-cezanne/leda-and-the-swan.jpg
Paul Cézanne	Le Vase rococo	Oil on Canvas	28 11/16 x 23 5/8 in.	1875-77	National Gallery of Art, Washington, D.C.	https://images.nga.gov/en/search/do_quick_search.htm
Paul Cézanne	Deux poires	Oil on Canvas	6 1/2 x 9 3/8 in.	1875	Christie's, New York	https://www.ecosia.org/images?addon=chrome&addonversion=
Paul Cézanne	Cinq pommes	Oil on Canvas	5 x 10 in.	1877-78	Mr. and Mrs. Eugene V. Thaw, New York	https://www.ecosia.org/images?addon=chrome&addonversion
Paul Cézanne	Pommes	Oil on Canvas	7 1/2 x 10 1/2 in.	1878	The Fitzwilliam Museum, Cambridge	https://uploads3.wikiart.org/images/paul-cezanne/apples-1878.jpg
Paul Cézanne	Pommes et linge	Oil on Canvas	9 13/16 x 17 5/16 in.	1879-80	Private Collection, Japan	http://www.cezannecatalogue.com/catalogue/entry.php?id=335
Paul Cézanne	Bol et boîte à lait	Oil on Canvas	7 7/8 x 7 1/16 in.	1879	Bridgestone Museum of Art, Tokyo	https://uploads2.wikiart.org/images/paul-cezanne/still-life-bowl-and-milk-
Paul Cézanne	Verre et pommes	Oil on Canvas	12 3/8 x 15 11/16 in.	1879-80	Collection Rudolf Staechelin, Basel	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Les Bégonias	Oil on Canvas	18 1/8 x 21 1/4 in.	1879-80	Private Collection	http://www.cezannecatalogue.com/catalogue_images/
Paul Cézanne	Deux fruits	Oil on Canvas	7 1/2 x 9 1/8 in.	1885	Private Collection, Japan	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Grenade et poires dans une assiette	Oil on Canvas	10 5/8 x 14 3/16 in.	1885	Private Collection, New York	https://uploads7.wikiart.org/images/paul-cezanne/
Paul Cézanne	Nature morte	Oil on Canvas	13 13/16 x 18 1/8 in.	1890	The State Hermitage Museum, St. Petersburg	https://uploads7.wikiart.org/images/paul-cezanne/still-life-with-apples-1.jpg
Paul Cézanne	Fruits et cruchon	Oil on Canvas	13 x 16 1/8 in.	1893-94	Museum of Fine Arts, Boston	http://www.mfa.org/collections/object/fruit-and-a-jug-on-a-table-33253
Paul Cézanne	La Corbeille de pommes	Oil on Canvas	25 5/8 x 31 1/2 in.	1893	The Art Institute of Chicago	https://uploads0.wikiart.org/images/paul-cezanne/
Paul Cézanne	Nature morte	Oil on Canvas	17 5/16	1895	Sammlung Esther	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52

			x 24 in.		Grether, Basel	
Paul Cézanne	La Baigneuse debout, s'essuyant les cheveux	Oil on Canvas	11 3/8 x 5 1/8 in.	1869	Private Collection	https://uploads1.wikiart.org/images/paul-cezanne/
Paul Cézanne	La Baignade	Oil on Canvas	7 1/2 x 10 5/8 in.	1875-77	Private Collection, New York	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Baigneur aux bras écartés	Oil on Canvas	13 x 9 3/8 in.	1877-78	Jasper Johns, Sharon, Conn.	https://uploads2.wikiart.org/images/paul-cezanne/
Paul Cézanne	Les Baigneurs au repos	Oil on Canvas	31 1/8 x 38 3/16 in.	1876-77	Barnes Foundation, Philadelphia (BF906)	https://uploads3.wikiart.org/images/paul-cezanne/bathers-at-rest-1877.jpg
Paul Cézanne	Quatre Baigneuses	Oil on Canvas	13 13/16 x 15 5/8 in.	1880	Private Collection	https://uploads0.wikiart.org/images/paul-cezanne/four-bathers-1878.jpg
Paul Cézanne	Baigneuses devant une tente	Oil on Canvas	24 13/16 x 33 1/16 in.	1883-85	Staatsgalerie, Stuttgart	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Le Baigneur au Rocher	Oil on Canvas	66 x 44 1/2 in. (part	1867-1869	Chrysler Art Museum Norfolk, VA	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Baigneuses	Oil on Canvas	13 x 15 11/16 in.	1870	Private Collection	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Homme Etendu et Baigneuses	Oil on Canvas	10 1/2 x 13 3/4 in.	1868-1870	Private Collection	Private Collection.
Paul Cézanne	Baigneur et Baigneuses	Oil on Canvas	7 7/8 x 15 3/4 in.	1870-1871	Travis Hanson, Fine Art, New York	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Baigneuse au bord de la mer	Oil on Canvas	9 1/4 x 8 5/8 in.	1875	Barnes Foundation, Philadelphia	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Baigneuse assise	Oil on Canvas	7 5/8 x 4 3/4 in.	1875	Columbus Museum of Art	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	La Baignade	Oil on Canvas	7 1/2 x 10 5/8 in.	1875-1877	Private Collection, New York	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Baigneur aux bras écartés	Oil on Canvas	9 1/2 x 5 1/2 in.	1876	Rose Art Museum, Brandeis University	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Baigneur aux bras écartés	Oil on Canvas	9 x 6 in.	1876-1877	Private Collection, France	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Cinq Baigneurs	Oil on Canvas	9 3/8 x 9 13/16 in.	1876	Musee d'Orsay, Paris France	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Baigneur aux bras écartés	Oil on Canvas	9 3/8 in. x 6 5/16 in.	1875-1876	Private Collection	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul	Baigneuses	Oil on	15 x	1875	The	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52

Cézanne		Canvas	18 1/8 in.		Metropolitan Museum of Art	
Paul Cézanne	Cinq Baigneuses sous des arbre	Oil on Canvas	23 5/8 x 28 11/16 in.	1875	Private Collection	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Trois Baigneuses	Oil on Canvas	7 1/2 x 8 11/16 in.	1874- 1875	Musee d'Orsay, Paris France	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Baigneurs au repose I	Oil on Canvas	15 x 18 in.	1875- 1876	Musee d'Art et d'Histoire, Geneva	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Baigneurs au repose II	Oil on Canvas	15 x 18 1/8 in.	1875- 1876	Location Unknown	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Baigneur debout	Oil on Canvas	11 13/16 in. x 6 11/16 in.	1876	Sotheby's New York	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Baigneur assis au bord de l'eau	Oil on Canvas	11 3/8 x 8 5/16 in.	1876	Obersteg Collection, Basel	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Trois Baigneuses	Oil on Canvas	9 3/8 x 12 1/2 in.	1876- 1877	Barnes Foundation, Philadelphia	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Trois Baigneuses	Oil on Canvas	9 13/16 x 13 1/5 in.	1876- 1877	Elliott Wolk, Scarsdale	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Trois Baigneuses	Oil on Canvas	20 1/2 x 21 3/8 in.	1876- 1877	Private Collection	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Trois Baigneuses	Oil on Canvas	12 x 13 in.	1875	Private Collections	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Quatre Baigneuses	Oil on Canvas	10 5/8 x 13 5/8 in.	1876- 1877	Barnes Foundation, Philadelphia	http://www.cezannecatalogue.com/exhibitions/entry.php?id=52
Paul Cézanne	Quatre Baigneuses	Oil on Canvas	15 x 18 1/8 in.	1877- 1878	Pola Museum of Art, Kanagawa	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Cinq Baigneuses	Oil on Canvas	15 1/2 x 16 1/2 in.	1877- 1878	Barnes Foundation, Philadelphia	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Cinq Baigneuses	Oil on Canvas	17 7/8 x 21 5/8 in.	1877- 1878	Musee Picasso, Paris	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Cinq Baigneuses	Oil on Canvas	15 x 16 1/8 in.	1877- 1878	Private Collection	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneuse	Oil on Canvas	8 5/8 x 5 1/2 in	1877- 1878	Kuboso Memorial Museum of Arts	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneur vu de dos	Oil on Canvas	9 3/8 in. x 7 1/2 in.	1877- 1878	Private Collection	http://www.cezannecatalogue.com/exhibitions

Paul Cézanne	Baigneur aux bras écartés	Oil on Canvas	28 3/4 x 23 5/8 in.	1877-1878	Private Collection	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Cinq Baigneurs	Oil on Canvas	13 5/8 x 15 in.	1879-1880	The Detroit Institute of Art	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Cinq Baigneurs	Oil on Canvas	23 5/8 x 28 3/4 in.	1880-1882	Private Collection	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneur	Oil on Canvas	12 5/8 x 8 5/8 in.	1877-1878	Private Collection	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Le Bain	Oil on Canvas	13 13/16 in. x 8 11/16 in.	1880-1882	Staatsgalerie, Stuttgart	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneur debout vu de dos	Oil on Canvas	13 in. x 8 11/16 in.	1879-1882	The Art Institute of Chicago	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneur debout vu de dos	Oil on Canvas	10 13/16 in. x 6 13/16 in.	1879-1882	Henry and Rose Pearlman Foundation	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Quatre Baigneuses	Oil on Canvas	13 13/16 in. x 15 5/8 in.	1880	Private Collection	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneurs et Baigneuses	Oil on Canvas	7 1/2 in. x 8 5/16 in.	1880	Ohara Museum, Kurashiki	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Cinq Baigneuses	Oil on Canvas	25 5/8 x 25 5/8 in.	1885	Kunst Musuem, Basel	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Le Grand Baigneur	Oil on Canvas	50 x 38 1/8 in.	1885	The Museum of Modern Art, New York	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Six Baigneuses also known as Les Ondines	Oil on Canvas	13 x 17 3/8 in.	1887	Private Collection, Geneva	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneurs	Oil on Canvas	23 5/8 x 31 7/8 in.	1890	Musee d'Orsay, Paris France	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneurs	Oil on Canvas	20 11/16 in. x 25 5/16 in.	1890-1892	Saint Louis Art Museum	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Quatre Baigneuses	Oil on Canvas	28 3/8 x 36 1/4 in.	1888-1890	Ny Carlsberg Glyptotek, Copenhagen	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneuses	Oil on Canvas	11 in. x 17 5/16	1890	Musee D'Orsay, Paris France	http://www.cezannecatalogue.com/exhibitions

			in.			
Paul Cézanne	Baigneuses	Oil on Canvas	11 1/4 x 20 in.	1890-1895	Stiftung Langmatt Sidney and Jenny Brown	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneurs	Oil on Canvas	15 3/8 x 20 7/8 in	1890	Private Collection	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneurs en plein air	Oil on Canvas	21 1/4 x 25 5/8 in.	1890-1891	The State Hermitage Museum, St. Petersburg	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneurs	Oil on Canvas	8 5/8 x 13 in.	1892	Musee de Beaux-Arts, Lyon	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneurs et Baigneuses	Oil on Canvas	8 7/8 x 13 15/16 in	1890	The Art Institute of Chicago	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Groupe de Baigneuses	Oil on Canvas	18 1/2 x 30 1/4 in.	1895	Ordrupgaard Collection, Copenhagen	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneurs	Oil on Canvas	7 1/2 x 10 1/4 in.	1890-1895	Private Collection	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Groupe de Baigneurs	Oil on Canvas	11 1/2 x 15 1/2 in.	1892-1894	Barnes Foundation, Philadelphia	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Le Bain	Oil on Canvas	10 1/4 x 15 3/4 in.	1892-1894	Pushkin Museum, Moscow	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Groupe de Baigneurs	Oil on Canvas	8 1/2 x 12 1/8 in.	1895	Philadelphia Museum of Art	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Les Grandes Baigneuses	Oil on Canvas	53 1/2 x 75 1/4 in.	1894-1905	National Gallery, London	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Les Grandes Baigneuses	Oil on Canvas	52 3/8 x 81 1/2 in.	1895-1906	Barnes Foundation, Philadelphia	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Les Grandes Baigneuses	Oil on Canvas	82 x 98 in.	1906	Philadelphia Museum of Art	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneuses	Oil on Canvas	11 x 14 1/4 in.	1900-1904	Private Collection	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneuses	Oil on Canvas	20 1/8 x 24 1/4 in	1899-1904	The Art Institute of Chicago	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Groupe de Sept Baigneurs	Oil on Canvas	14 5/8 x 17 3/4 in.	1900	Fondation Beyeler, Riehen, Basel	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneurs	Oil on Canvas	10 5/8 x 18 1/4 in.	1898-1900	The Baltimore Museum of Art	http://www.cezannecatalogue.com/exhibitions
Paul	Baigneurs	Oil on	8 5/8	1899-	Musee	http://www.cezannecatalogue.com/exhibitions

Cézanne		Canvas	x 13 in.	1900	d'Orsay, Paris France	
Paul Cézanne	Baigneurs	Oil on Canvas	10 5/8 x 8 5/8 in	1900	Location Unknown	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneurs	Oil on Canvas	16 5/8 x 21 5/8 in.	1900-1904	Musee d'Orsay, Paris France	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneurs	Oil on Canvas	11 7/8 x 17 3/8 in.	1900	Location Unknown	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Esquisse de Baigneurs	Oil on Canvas	7 7/8 x 13 in.	1900-1902	Private Collection	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneuses	Oil on Canvas	11 1/2 x 9 1/4 in.	1902-1906	Private Collection	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Esquisse de Baigneurs	Oil on Canvas	12 5/8 x 15 3/4 in.	1902-1906	Private Collection	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Esquisse de Baigneurs	Oil on Canvas	25 5/8 x 31 7/8 in.	1900-1906	Fondazione Magnani-Rocca, Parma	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneuses	Oil on Canvas	13 3/4 x 8 5/8 in.	1900	Private Collection	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Esquisse de Baigneuses	Oil on Canvas	9 x 6 5/8 in.	1900	Becon Collection, Greenwich	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneurs	Oil on Canvas	9 x 10 1/2 in.	1902-1904	Barnes Foundation, Philadelphia	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Etude de Baigneuses	Oil on Canvas	8 3/8 x 12 3/4 in.	1900-1906	Museum Rosengart, Lucerne	http://www.cezannecatalogue.com/exhibitions
Paul Cézanne	Baigneuses	Oil on Canvas	29 x 36 3/8 in.	1902-1906	Private Collection	http://www.cezannecatalogue.com/exhibitions
Paolo Veronese	Les Noces de Cana (before conservation)	Oil on Canvas	22 ft.3 in. x 32 ft	1563	Louvre (unrestored)	https://www.wikiart.org/en/paolo-veronese/the-marriage-at-cana-
Paolo Veronese	Les Noces de Cana (digital facsimile)	Digital Reproduction	22 ft. 3 in. x 32 ft.	2006	Fondazione Giorgio Cini, Venice	https://www.cineclubdecaen.com/peinture/peintres/veronese/nocesdecana.jpg

Table 2.

SUPPLEMENTARY TABLE II: Golden Windows

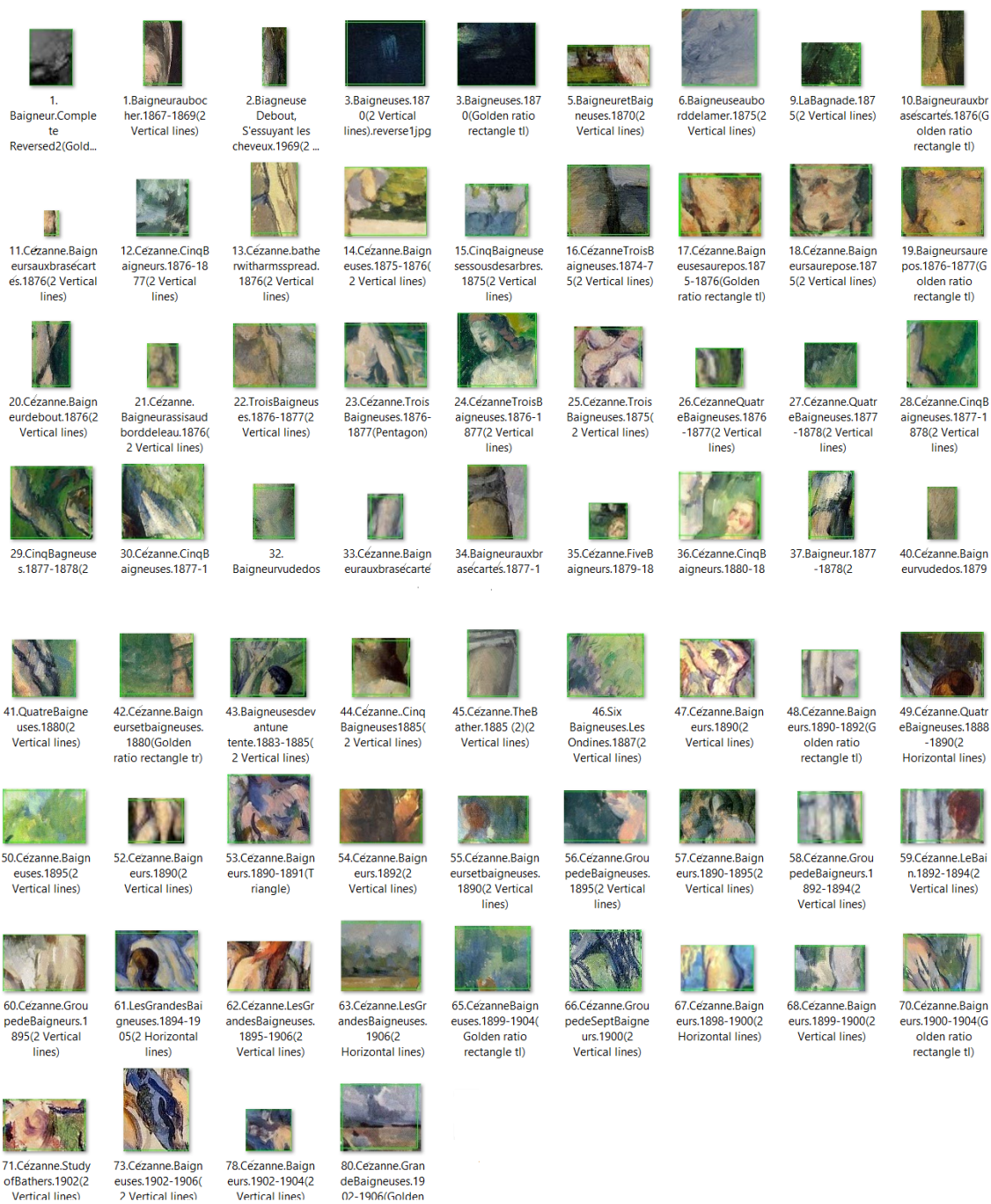


Figure 19. SUPPLEMENTARY TABLE II

SUPPLEMENTARY TABLE III: Golden Strips

Figure 20. SUPPLEMENTARY TABLE III

Notes

- [1] In *A New History of the Humanities: The Search for Principles and Patterns from Antiquity to the Present* (Oxford: Oxford University Press, 2013), Rens Bod maps art history's trajectory in the western world. For Bod, the formal study of art history begins with Vitruvius's theory of harmonious proportion in *On Architecture* (15 BCE) and continues with Pliny's discussions of illusionism in *Natural History* (77 CE). Though China had a thriving field of art historical scholarship in the Middle Ages, the western world produced few written discussions of art between Pliny and Alberti's works. Bod cites Alberti's *On Painting* (1435) as the first theoretical treatise on art, followed by Piero della Francesca's "On the Perspective for Painting" (1435), A Treatise on Painting, attributed to Leonardo da Vinci and compiled by Francesco Melzi (1540), then Vasari's *Lives of the Most Excellent Painters, Sculptors and Architects* (1550). Bod views the advent of the digital humanities as a natural extension of the general search for principles and patterns that directs humanistic inquiry. 311-325.
- [2] Kugler divided art history into pre-Greek, classical, romantic and modern periods. Burckhardt identified the features of Renaissance art. Wölfflin distinguished Baroque from Renaissance art through the relations of differences principle employed in structural linguistics. In analytically comparing paintings, Wölfflin showed that Renaissance art is "linear, flat, and closed," while Baroque art is "painterly, deep, and open."
- [3] Wikipedia contributors, "Heinrich Wölfflin," Wikipedia, The Free Encyclopedia, (accessed September 13, 2017). https://en.wikipedia.org/wiki/Heinrich_W%C3%B6lfflin.
- [4] Bod, p. 320. "No matter how much the digital, computational approach is in its infancy, it provides a powerful tool for testing hypotheses relating to paintings."
- [5] Matthew Battles and Michael Maizels, "Collections and/of Data: Art History and the Art Museum in the DH Mode" in *Debates in the Digital Humanities* (2016). <http://dhdebates.gc.cuny.edu/debates/2>.
- [6] See also Vols. 7531 (2010) and 7869 (2011).
- [7] The New York Times announced that The Metropolitan Museum is making 375,000 images from its collections available online "for however you want to use them." The National Gallery of Art in Washington, The Rijksmuseum in Amsterdam, and the Museum of Modern Art in New York have also made their collections freely available to the public for downloading. Joshua Baron, "MET Museum Makes 350,000 Images Free." *New York Times*, February 7, 2017.
- [8] "It should be difficult today to conceive of the computer as any more than a speedy idiot obeying a set of precise instructions; but many in the humanities do indeed continue to regard it as a black and magical (or black-magical) box — throw the data at it, and somehow the computer will make sense of them." Michael Greenhalgh, *A Companion to Digital Humanities*. <http://www.digitalhumanities.org/companion/>. Stephen Ramsay in *Humane Computation*, summarizes objections to DH: "There are many who think scatter plots filled with data points drawn from, say, English novels, are already a crime against the humanities — the death of all that is good and pure about humanistic study. For them, the problem is positivism in its properly technical sense. They fear an epistemology that does not merely value empirical data, but which (in its extreme philosophical forms) considers empirical data to be the only valid form of evidence. They imagine a computationally driven history or French literature curriculum that forsakes the ancient circle of the seminar for the modern angles of the server room. They imagine humanistic conversation debasing itself in the form of technical caviis, humanistic ethics becoming nothing more than 'practical business ethics,' and teaching degenerating into mere training."
- [9] www.sciencedaily.com/releases/2019/01/190107131236.html. Accessed 1/9/19.
- [10] In the context of book history, and the shift from orality to literacy, Marshall McLuhan discusses the way technology "extends" the senses in *The Gutenberg Galaxy: The Making of Typographic Man*. Toronto: University of Toronto Press, 1965. 40.
- [11] J.T. Clark's famous "Cézanne's Freud". *Representations* 52. Fall 1995 is representative of this approach. Of Cézanne's *Baigneurs en repose* he writes, "never, for a start, has a picture declared itself so openly — so awkwardly — made out of separate, overdetermined parts coexisting only in sufferance." 94.
- [12] "The practices include the embrace of minimal computing, small data sets, local archives, and freely available platforms for creating small-scale digital humanities projects while working with undergraduate students." Roopika Risam and Susan Edwards, "Micro DH: Digital Humanities at the Small Scale" <https://dh2017.adho.org/abstracts/196/196.pdf>.
- [13] In "Distant Reading and Intellectual History", Ted Underwood writes that distant reading "doesn't overemphasize technology, and it candidly admits that new methods are mainly useful at larger scales of analysis." Our work here scales down a distant reading approach. 138 paintings is still more than the brain can handle in making comparisons, nevertheless, the data set is small enough to allow the researcher to toggle between the images and graphical output to verify hypotheses and make discoveries.
- [14] Can digital images replace physical paintings as serious objects for study? Certainly, there are irreducible elements to paintings that do not translate well electronically or in print, and that demand first-person viewing, such as the luminous and changing effects of varnishes and glazes on paintings, and the layering and aging of paints. Whether in print or online, all reproductions of paintings are "approximations". In even the most expensively illustrated books, the same paintings may vary in color. Phillip Ball discusses the limitations of print and the digital reproductions of works of art in *Bright Earth*. He observes how "for a coloristically complex picture like Danae, no two reproductions will look the same." Nevertheless, "All written discourses of art must rely on reproductions to make their point." Art Historians depend on approximations for their work, and computer-assisted systems, though they convert these approximations into numbers, can provide us with data that is inaccessible subjectively, data that the researcher must interpret.
- [15] The Louvre did not acquisition Ingres' *La Source* until 1878, nine years after *Baigneuse debout, s'essuyant les cheveux* was painted. Nevertheless, the painting was exhibited in the official Paris Salon in 1856 and was famous when Cézanne was a young student in 1861.
- [16] On Cézanne's famous statement that he wished to bring "Poussin back to nature" see Richard Shiff, "Introduction," *Conversations with Cézanne*, xxi. His footnote, p. 217 provides valuable bibliography on this topic.
- [17] Bruno Latour and Adam Lowe have discussed of the painting's history as a material object and as a facsimile.

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